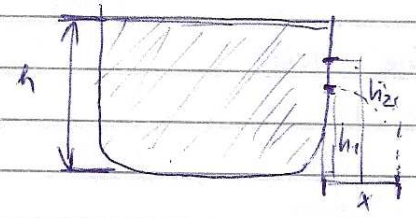
4.6. / nevieš. Nádoba valcového tvaru – 2 otvory vo výškach h_1 a h_2
v akej výške musí byť hladina, aby vzdialenosť z oboch otvorov do miesta dopadu bola rovnaká?



Torricelliho norec: $v = \sqrt{2gh}$ (h = výšk. diera od hladiny)

vodorovný vrh:

$x = v \cdot t$

$y = h - \frac{1}{2} g t^2$

pri dopade $y = 0 \Rightarrow h = \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

$x_1 = v_1 t_1 \qquad t_1 = \sqrt{\frac{2h_1}{g}} \qquad v_1 = \sqrt{2g(h-h_1)}$

$x_2 = v_2 t_2 \qquad t_2 = \sqrt{\frac{2h_2}{g}} \qquad v_2 = \sqrt{2g(h-h_2)}$

$x_1 = x_2$

$v_1 \cdot t_1 = v_2 t_2$

$$\sqrt{2g(h-h_1)} \cdot \sqrt{\frac{2h_1}{g}} = \sqrt{2g(h-h_2)} \cdot \sqrt{\frac{2h_2}{g}} \quad /()^2$$

$$(2gh - 2gh_1) \cdot \frac{2h_1}{g} = (2gh - 2gh_2) \cdot \frac{2h_2}{g} \quad / \cdot g$$

$$4ghh_1 - 4gh_1^2 = 4ghh_2 - 4gh_2^2 \quad / : 4g$$

$$hh_1 - h_1^2 = hh_2 - h_2^2$$

$$hh_1 - hh_2 = h_1^2 - h_2^2$$

$$h(h_1 - h_2) = (h_1 - h_2)(h_1 + h_2)$$

$$\underline{h = h_1 + h_2}$$

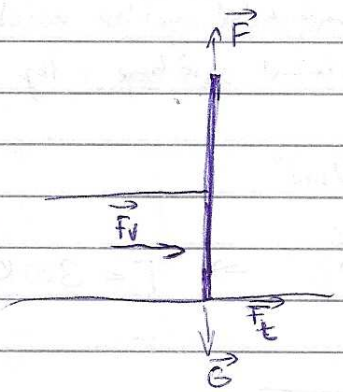
4.2 Akú F na dvihnutie hrádze?

$m_{hrádze} = 250\,g$

šírka $b = 3\,m$

hĺbka $h = 1{,}5\,m$

koeficient trenia ocele $\mu = 0{,}3$



$$F = G + F_t =$$

$$= m \cdot g + \mu \cdot F_v$$

$$p = \frac{dF_v}{dS} \Rightarrow dF_v = p\,dS$$

$$dF_v = x \cdot \rho \cdot g \cdot b \cdot dx$$

$$F_v = \rho \cdot g \cdot b \int_0^h x\,dx = \rho \cdot g \cdot b \left[\frac{x^2}{2}\right]_0^h =$$

$$= \frac{1}{2} \rho \cdot g \cdot b \cdot h^2$$

$$F = m \cdot g + \mu \cdot \frac{1}{2} \rho \cdot g \cdot b \cdot h^2 =$$

$$= 250\,kg \cdot 9{,}81\,m \cdot s^{-2} + 0{,}3 \cdot \frac{1}{2} \cdot 10^3\,kg\,m^{-3} \cdot 9{,}81\,m \cdot s^{-2} \cdot 3\,m \cdot (1{,}5\,m)^2 =$$

$$= \underline{12\,385{,}125\,N}$$

5.8 Střední kvadratická rychlost molekul plynu je $v_s = 800\,m\,s^{-1}$. Kolik molekul obsahuje 1 kg tohoto plynu při 27 °C?

$v_s = 800\,m\,s^{-1}$

$m = 1\,kg$

$t = 27\,°C \Rightarrow T = 300\,K \quad (t + 273 = T)$

$N = ?$

↳ počet

$$v_s = \sqrt{\frac{3kT}{m_0}} \Rightarrow v_s^2 = \frac{3kT}{m_0}$$

$$m_0 = \frac{3kT}{v_s^2}$$

$$N = \frac{m}{m_0} = \frac{m \cdot v_s^2}{3kT} = \frac{1\,kg \cdot (800\,m\,s^{-1})^2}{3 \cdot 1{,}38 \cdot 10^{-23}\,J\,K^{-1} \cdot 300\,K} = 5{,}15 \cdot 10^{25}$$

5.1 Bomba obsahuje při $t_1 = 27\,°C = T = 300\,K$ a tlaku $p_1 = 4\,MPa$ stlačený plyn. Jaký tlak $p_2 = ?$ ukáže 1/2 plynu vypuštěna a $t_2 = 27° - 15° = 12\,°C \Rightarrow T = 285\,K$

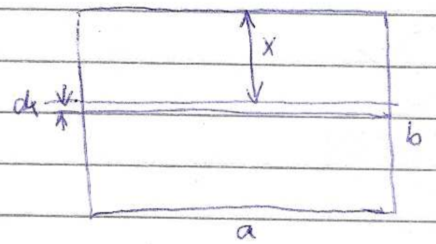
(1) $p_1 \cdot V = \frac{m_1}{M} R T_1$

(2) $p_2 \cdot V = \frac{m_2}{M} R T_2 = \frac{m_1}{2M} R T_2$

$$\frac{(2)}{(1)} \quad \frac{p_2 V}{p_1 V} = \frac{\frac{m_1}{2M} \cdot R T_2}{\frac{m_1}{M} R T_1} = \frac{T_2}{2T_1}$$

$$p_2 = p_1 \cdot \frac{T_2}{2T_1} = 4 \cdot 10^6\,Pa \cdot \frac{285\,K}{2 \cdot 300\,K} = 1{,}9 \cdot 10^6\,Pa = 1{,}9\,MPa$$

4.2) Akvárium: Akou silou pôsobí voda na bočnú stenu akvária $a = 0,6\,m \times 0,4\,m$ ak je naplnené až po okraj.



$$p = \frac{dF}{dS} \Rightarrow dF = p \cdot dS = x \rho g \cdot a\,dx$$

$$F = \rho \cdot g a \int_0^b x\,dx = \rho g a \left[\frac{x^2}{2}\right]_0^b = \frac{1}{2} \rho g a b^2$$

$$F = \frac{1}{2} \cdot 10^3\,m\,kg^{-3} \cdot 9,81\,m\,s^{-2} \cdot 0,6\,m \cdot (0,4\,m)^2 = \underline{\underline{470,9\,N}}$$

4.10) Striekačka:

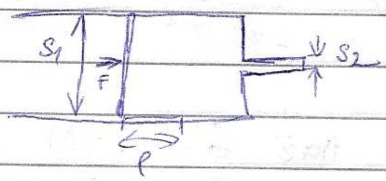
$S_1 = 1\,cm^2 = 10^{-4}\,m^2$

$S_2 = 1\,mm^2 = 10^{-6}\,m^2$

$F = 4,9\,N$

$l = 4,9\,cm = 0,049\,m$

$t = ?$



$$l = v \cdot t \Rightarrow t = \frac{l}{v_1} \quad ; \quad v_1 = ?$$

$$p_1 + \frac{1}{2} \rho v_1^2 = p_2 + \frac{1}{2} \rho v_2^2$$

$$\frac{F}{S_1} + p_A + \frac{1}{2} \rho v_1^2 = p_A + \frac{1}{2} \rho v_2^2 \quad ; \quad S_1 v_1 = S_2 v_2 \Rightarrow v_2 = \frac{S_1 v_1}{S_2}$$

$$\frac{F}{S_1} + \frac{1}{2} \rho v_1^2 = \frac{1}{2} \rho \cdot \frac{S_1^2 v_1^2}{S_2^2} \quad \Big/ - \frac{1}{2} \rho v_1^2$$

$$\frac{F}{S_1} = \frac{1}{2} \rho v_1^2 \left(\frac{S_1^2}{S_2^2} - 1\right)$$

$$v_1 = \sqrt{\frac{2F}{\rho S_1 \left(\frac{S_1^2}{S_2^2} - 1\right)}}$$

$$t = \frac{l}{v_1} = l \cdot \sqrt{\frac{\rho S_1 \left(\frac{S_1^2}{S_2^2} - 1\right)}{2F}} = 0,049 \cdot \sqrt{\frac{10^3 \cdot 10^{-4} \cdot \left(\frac{10^{-8}}{10^{-12}} - 1\right)}{2 \cdot 4,9}} \doteq \underline{0,49\,s}$$

5.5 Aký objem zaujíma 4 g hélia pri tlaku 99 991,5 Pa a teplote 20 °C? Aká je hustota plynu?

$m = 4\,g = 4 \cdot 10^{-3}\,kg$
$p = 99\,991{,}5\,Pa$
$t = 20\,°C \Rightarrow T = 293\,K$
$M = 4 \cdot 10^{-3}\,kg\,mol^{-1}$
$V = ?$
$\rho = ?$

$$pV = \frac{m}{M} RT$$

$$V = \frac{mRT}{Mp} = \frac{4 \cdot 10^{-3} \cdot 8{,}314 \cdot 293}{4 \cdot 10^{-3} \cdot 99\,991{,}5} \doteq 243 \cdot 10^{-3}\,m^3 = 24{,}3\,l$$

$$\rho = \frac{m}{V} = \frac{4 \cdot 10^{-3}}{243 \cdot 10^{-3}} = 0{,}16\,kg \cdot m^{-3}$$

4.3 sklo tiaž $G = 1{,}37\,N$
$G_z = 0{,}824\,N$
$\rho_s = ?$

ρ_s – hustota skla
ρ_v – hustota vody

$$\rho_s = \frac{m}{V}$$

vztlaková sila $F_{vz} = m \cdot s \cdot \rho_v \cdot V$

$$V = \frac{F_{vz}}{\rho \cdot g}$$

$$F_{vz} = G_1 - G_z$$

tiaž: $G = m \cdot g \Rightarrow m = \frac{G_1}{g}$

$$\rho_s = \frac{m}{V} = \frac{\frac{G_1}{g}}{\frac{F_{vz}}{\rho \cdot g}} = \frac{G_1}{g} \cdot \frac{\rho_v \cdot g}{F_{vz}} = \frac{G_1 \rho_v}{F_{vz}} =$$

$$= \frac{1{,}37 \cdot 10^3}{0{,}546} = 2.509\,kg \cdot m^{-3}$$