Fyzika I. II. Riešené príklady

Obsah

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Veľmi ma potešíte ak vám tento dokument pomôže a ešte viac ak budete stránku zdieľať na facebooku. ☺

1. Kinematika hmotného bodu

Bod sa pohybuje priamočiaro tak, že jeho dráha závisí od času $X=At+Bt^2$, $A = 5\ m.s^{-1}$ $B= 6\ m.s^{-1}$ Aká je priemerná rýchlosť v_p v časovom intervale medzi deviatou a dvanástou sekundou, aké sú okamžité rýchlosti a zrýchlenia v týchto dvoch okamihoch ?

$$X = At + Bt^2$$

$$A = 5\ m\,s^{-1}$$

$$B = 6\ m\,s^{-2}$$

$$v_p,\ v_9,\ v_{12},\ a_9,\ a_{12} = ?$$

$$\boxed{\vec{v} = \frac{d\vec{s}}{dt}} \qquad \vec{X} = \vec{s}_x$$

$$\vec{v}_x = \vec{i}\,\frac{dx}{dt} = \vec{i}\,\frac{d}{dt}\left(At + Bt^2\right) = (A + 2Bt)\,\vec{i}$$

$$\vec{v}_x = (A + 2Bt)\,\vec{i}$$

$$v_9 = 5 + 2.9.9 = \underline{\underline{113\ m\,s^{-1}}}$$

$$v_{12} = 5 + 2.6.12 = \underline{\underline{149\ m\,s^{-1}}}$$

$$v_p = \frac{v_9 + v_{12}}{2} = \frac{113 + 149}{2} = \underline{\underline{131\ m\,s^{-1}}}$$

$$\boxed{\vec{a}_x = \frac{d\vec{v}}{dt}} = \frac{d}{dt}(A + 2Bt)\,\vec{i} = 2B\,\vec{i}$$

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$$a_x = 2 . B = 2 . 6 = \underline{\underline{12\, m\, s^{-2}}}$$

Teleso sa pohybuje tak , že jeho dráha závisí od času podľa vzťahu $s = A + Bt + Ct^2 + Dt^3$ Kde $C = 0{,}14 m.s^{-2}$ $D = 0{,}01$ $m.s^{-3}$ V akom časovom okamžiku t_1 od začiatku pohybu bude mať zrýchlenie telesa hodnotu $a_1 = 1 m.s^{-2}$? Učte priemerné zrýchlenie v časovom intervele od začiatku pohybu po čas t_1 .

$$s = A + Bt + Ct^2 + Dt^3$$

$$C = 0{,}14\, m\, s^{2}$$

$$D = 0{,}01\, m\, s^{3}$$

$$a_1 = 1\, m\, s^{2}$$

$$t_1 = ?$$

$$a_p = ?$$

$$\vec{v} = \frac{d\vec{s}}{dt} = \frac{d}{dt}\left(A + Bt + Ct^2 + Dt^3\right)\vec{i} =$$

$$= \left(B + 2Ct + 3Dt^2\right)\vec{i} \qquad \boxed{\vec{v} = \left(B + 2Ct + 3Dt^2\right)\vec{i}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(B + 2Ct + 3Dt^2\right)\vec{i} = \left(2C + 6Dt\right)\vec{i}$$

$$\boxed{\vec{a} = \left(2C + 6Dt\right)\vec{i}}$$

$$a_1 = 2C + 6Dt_1$$

$$\frac{a_1 - 2C}{6D} = t_1 = \frac{1 - 2 \cdot 0{,}14}{6 \cdot 0{,}01} = \underline{\underline{12\,s}}$$

$\vec{a_0}$ $t_0 = 0\,s$

$$\vec{a_0} = (2C - 6Dt)\vec{i} = 2C = 2 \cdot 0{,}14 = 0{,}28\ m\,s^{-2}$$

$t_1 = 12\,s$

$$\vec{a_p} = \frac{\vec{a_0} + \vec{a_1}}{2} = \frac{0{,}28 + 1}{2} = \underline{\underline{0{,}64\ m\,s^{-2}}}$$

Závislosť dráhy hmotného bodu od času je daná rovnicou $s = At - Bt^2 + Ct^3$ Kde $A = 2\ m.s^{-1}$ $B = 3\ m.s^{-2}$ $C = 4\ m.s^{-3}$ Vyjadríte rýchlosť a zrýchlenie hmotného bodu ako funkciu času. Vypočítajte rýchlosť a dráhu v časovom okamihu $t_1 = 2\ s$

$$\vec{s} = (At - Bt^2 + Ct^3)\vec{i}$$

$A = 2\,m\,s^{-1}$

$B = 3\,m\,s^{-2}$

$C = 4\,m\,s^{-3}$

$v_1 = ?$ $t_1 = 2\,s$

$$\boxed{\vec{v} = \frac{d\vec{s}}{dt}} = \frac{d}{dt}(At - Bt^2 + Ct^3)\vec{i}$$

$$\underline{\underline{\vec{v} = A - 2Bt + 3Ct^2}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(A - 2Bt + 3Ct^2\right)\vec{i}$$

$$\vec{a} = (2B + 6Ct)\,\vec{i}$$

$t_1 = 2s$

$$a_1 = 2B + 6Ct_1 = 2 \cdot 3 + 6 \cdot 4 \cdot 2 = 54\ m s^{-2}$$

$t_1 = 2s$

$$v_1 = A - 2Bt + 3Ct^2 = 2 - 2 \cdot 3 \cdot 2 + 3 \cdot 4 \cdot 2^2 = \underline{\underline{38\ m s^{-1}}}$$

$t_1 = 2s$

$$s_1 = At - Bt^2 + Ct^3 = 2 \cdot 2 - 3 \cdot 2^2 + 4 \cdot 2^3 = \underline{\underline{24\ m}}$$

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Teleso sa pohybuje priamočiaro. Dráha závisí od rýchlosti podľa vzťahu $s = A + Bt - Ct^2$
Vypočítajte rýchlosť a zrýchlenie v okamihoch $t_1 = 0s$ $t_2 = 3s$ $t_3 = 5s$ a nakreslite graf závislosti rýchlosti a zrýchlenia od času ak, $A = 4$ m $B = 40$ m.s^{-1} $C = 4$ m.s^{-2}

$s = A + Bt - Ct^2$

$t_1 = 0s$ $t_2 = 3s$ $t_3 = 5s$

$A = 4m$

$B = 40 m s^{-1}$

$C = 4 m s^{-2}$

$v_1, v_2, v_3, a_1, a_2, a_3 = ?$

$$\boxed{v = \frac{ds}{dt}} = \frac{d}{dt}(A + Bt - Ct^2) = B - 2Ct$$

$v_1, t_1 = 0s$ $v_1 = B - 2Ct_1 = 40 - 2.4.0 = \underline{\underline{40 m s^{-1}}}$

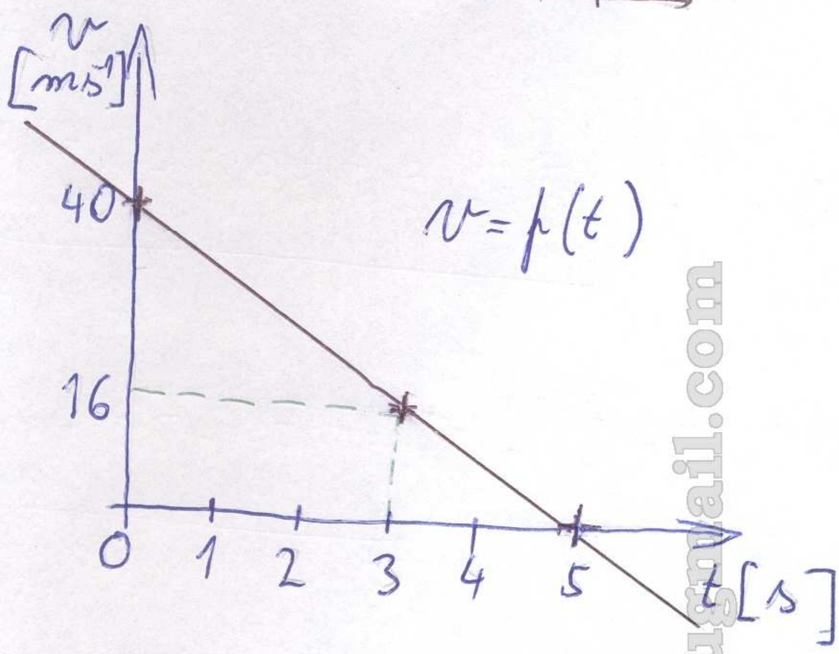
$v_2, t_2 = 3s$ $v_2 = B - 2Ct_2 = 40 - 2.4.3 = \underline{\underline{16 m s^{-1}}}$

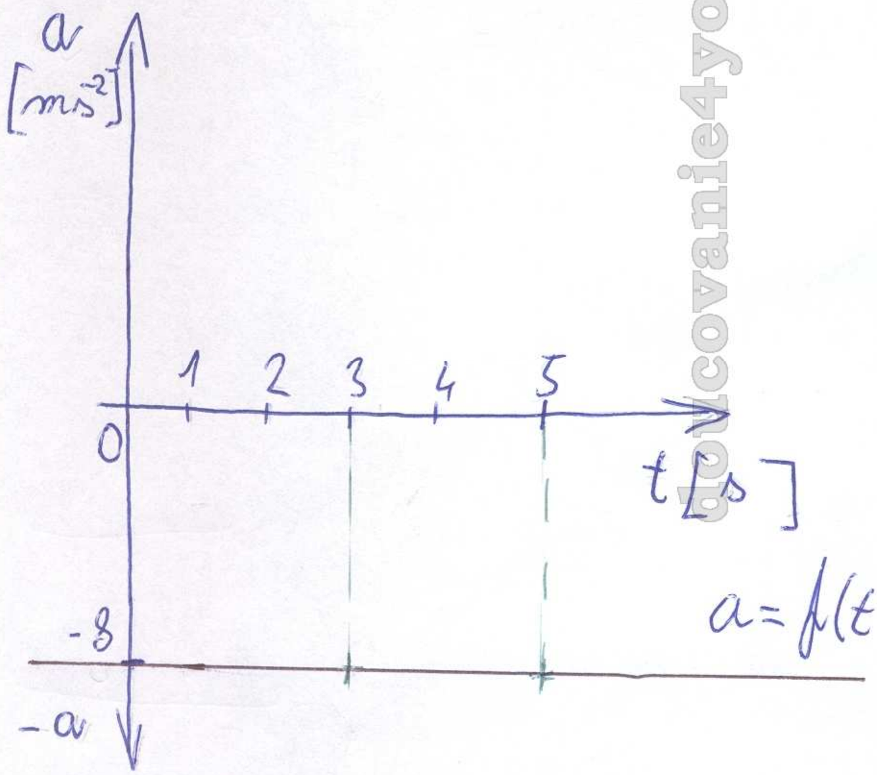
$v_3, t_3 = 5s$ $v_3 = B - 2Ct_3 = 40 - 2.4.5 = \underline{\underline{0 m s^{-1}}}$

$$\boxed{a = \frac{dv}{dt}} = \frac{d}{dt}(B - 2Ct) = -2C$$

$a = konšt \Rightarrow a_1 = a_2 = a_3 = -2C = -2.4 = \underline{\underline{-8 m s^{-2}}}$

$t\,[s]$	0	3	5
$v\,[m\,s^{-1}]$	40	16	0
$a\,[m\,s^{-2}]$	-8	-8	-8





Pohyb bodu je určený rovnicami $X = A_1t^2 + B_1$ $Y = A_2t^2 + B_2$ Kde $A_1 = 20 cm.s^{-2}$ $B_1 = 5cm$ $A_2 = 15cm\ s^{-2}$ $B_2 = -3cm$ Vypočítajte veľkosť a smer rýchlosti a zrýchlenia v čase $t_1 = 2s$

$x = A_1 t^2 + B_1$

$y = A_2 t^2 + B_2$

$A_1 = 20\ cm\ s^{-2}$

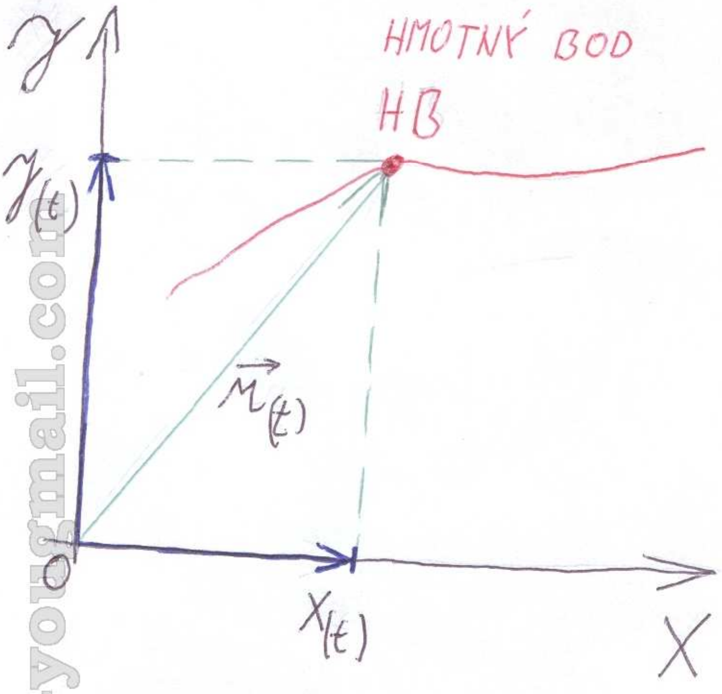
$B_1 = 5\ cm$

$A_2 = 15\ cm\ s^{-2}$

$B_2 = -3\ cm$

$t_1 = 2\ s$

$v_1, a_1, \cos\alpha = ?$



$x_{(t)} = A_1 t^2 + B_1$

$y_{(t)} = A_2 t^2 + B_2$

$$\boxed{v_x = \frac{dx}{dt}} = \frac{d}{dt}(A_1 t^2 + B_1) = 2A_1 t = 2.20.2 = 80\ cm\ s^{-1}$$

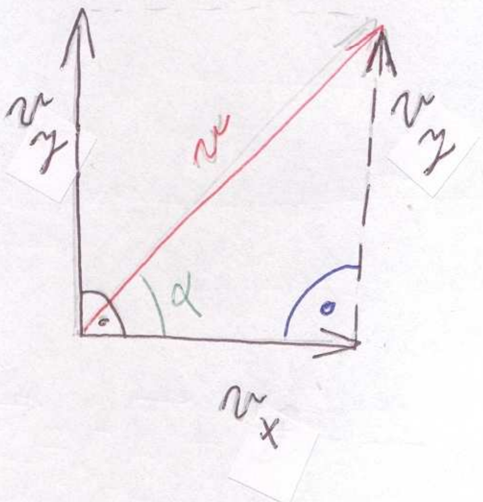
$$\boxed{v_y = \frac{dy}{dt}} = \frac{d}{dt}(A_2 t^2 + B_2) = 2A_2 t = 2.15.2 = 60\ cm\ s^{-1}$$

$$\boxed{a_x = \frac{dv_x}{dt}} = \frac{d}{dt}(2A_1 t) = 2A_1 = 2.20 = 40\ cm\ s^{-2}$$

$$\boxed{a_y = \frac{dv_y}{dt}} = \frac{d}{dt}(2A_2 t) = 2A_2 = 2.15 = 30\ cm\ s^{-2}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{80^2 + 60^2} = 100\,cm\,s^{-1} = \underline{\underline{1\,m\,s^{-1}}}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{40^2 + 30^2} = 50\,cm\,s^{-2} = \underline{\underline{0{,}5\,m\,s^{-2}}}$$



$\Longrightarrow$ TO ISTÉ PRE $\underline{a}$

$$\cos\alpha = \frac{v_x}{v} = \frac{80}{100} = \underline{\underline{0{,}8}}$$

$$\sin\alpha = \frac{v_y}{v} = \frac{60}{100} = 0{,}6$$

$$\alpha = \arccos 0{,}8 = 36{,}87^\circ$$

Častica sa pohybuje tak, že jej poloha v ľubovoľnom okamihu je určená polohovým vektorom $\vec{r} = t.\vec{i} + \left(t + \frac{t^2}{2}\right).\vec{j} - \frac{4}{\pi^2}.\left\{\sin\frac{\pi . t}{2}\right\}.\vec{k}$ Učte veľkosť rýchlosti a zrýchlenia častice v čase $t_1 = 1s$

$$\vec{r} = t\vec{i} + \left(t + \frac{t^2}{2}\right)\vec{j} - \frac{4}{\pi^2}\left(\sin\frac{\pi t}{2}\right)\vec{k} \ (m)$$

$t = 1s$

$\vec{v}_1, \vec{a}_1 = ?$

$$\vec{x} = t\vec{i}$$

$$\vec{y} = \left(t + \frac{t^2}{2}\right)\vec{j}$$

$$\vec{z} = -\frac{4}{\pi^2}\left(\sin\frac{\pi t}{2}\right)\vec{k}$$

$$\boxed{\vec{v}_x = \frac{d\vec{x}}{dt}} = \frac{d}{dt}(t)\vec{i} = 1\vec{i} = 1\,ms^{-1}$$

$$\boxed{\vec{v}_y = \frac{d\vec{y}}{dt}} = \frac{d}{dt}\left(t + \frac{t^2}{2}\right)\vec{j} = (1+t)\vec{j} = 1+1 = 2\,ms^{-1}$$

$$\boxed{\vec{v}_z = \frac{d\vec{z}}{dt}} = \frac{d}{dt}\left(-\frac{4}{\pi^2}\left(\sin\frac{\pi t}{2}\right)\right)\vec{k} =$$

$$= -\left(\frac{4}{\pi^2}\left(\cos\frac{\pi t}{2}\right)\frac{\pi}{2}\right)\vec{k} = -\frac{2}{\pi}\cos\frac{\pi t}{2}$$

$$\vec{v}_z = -\left(\frac{2}{\pi}\cos\frac{\pi t}{2}\right)\vec{k} = -\frac{2}{\pi}\cos\frac{\pi\cdot 1}{2} = -0{,}45$$

$$\boxed{\vec{a}_x = \frac{d\vec{v}_x}{dt}} = \frac{d}{dt}\left(\vec{i}\right) = 0\,m\,s^{-2}$$

$$\boxed{\vec{a}_y = \frac{d\vec{v}_y}{dt}} = \frac{d}{dt}\left(1+t\right)\vec{j} = 1\cdot\vec{j} = 1\,m\,s^{-2}$$

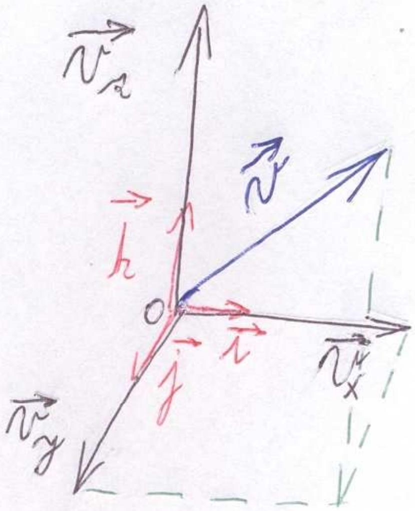
$$\boxed{\vec{a}_z = \frac{d\vec{v}_z}{dt}} = \frac{d}{dt}\left(\frac{-2}{\pi}\cos\frac{\pi t}{2}\right)\vec{k} =$$

$$= -\frac{\cancel{2}}{\cancel{\pi}}\left(-\sin\frac{\pi t}{2}\right)\frac{\cancel{\pi}}{\cancel{2}}\vec{k} = \left(\sin\frac{\pi t}{2}\right)\vec{k}$$

$$\vec{a}_z = \left(\sin\frac{\pi t}{2}\right)\vec{k} = \sin\frac{\pi\cdot 1}{2} = 0{,}707\,m\,s^{-2}$$

$$\boxed{|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}} = \sqrt{1^2 + 2^2 + (-0{,}45)^2} = \underline{\underline{2{,}28\,m\,s^{-1}}}$$

$$\boxed{|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}} = \sqrt{0^2 + 1^2 + 0{,}707^2} = \underline{\underline{1{,}22\,m\,s^{-2}}}$$



$$\Rightarrow |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

ALTERNATIVNY ZÁPIS

$$\vec{r} = t\vec{i} + \left(t + \frac{t^2}{2}\right)\vec{j} - \frac{4}{\pi^2}\left(\sin\frac{\pi t}{2}\right)\vec{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \boxed{\frac{d}{dt}}\left[t\vec{i} + \left(t + \frac{t^2}{2}\right)\vec{j} - \frac{4}{\pi^2}\left(\sin\frac{\pi t}{2}\right)\vec{k}\right] =$$

$$= \boxed{\frac{d}{dt}}\left(t\vec{i}\right) + \boxed{\frac{d}{dt}}\left(\left(t + \frac{t^2}{2}\right)\vec{j}\right) - \boxed{\frac{d}{dt}}\left(\frac{4}{\pi^2}\left(\sin\frac{\pi t}{2}\right)\vec{k}\right)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

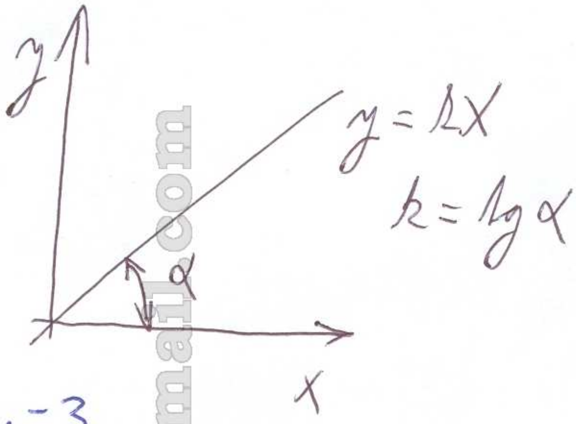
Rušeň sa rozbieha priamočiaro z pokoja s rovnomerne rastúcim zrýchlením, tak že v čase $t_1 = 100s$ ma zrýchlenie hodnotu $a_1 = 0{,}5 m.s^{-2}$ Vypočítajte rýchlosť v čase t_1 a dráhu ktorú za ten čas prešiel.

$t_1 = 100\,s$

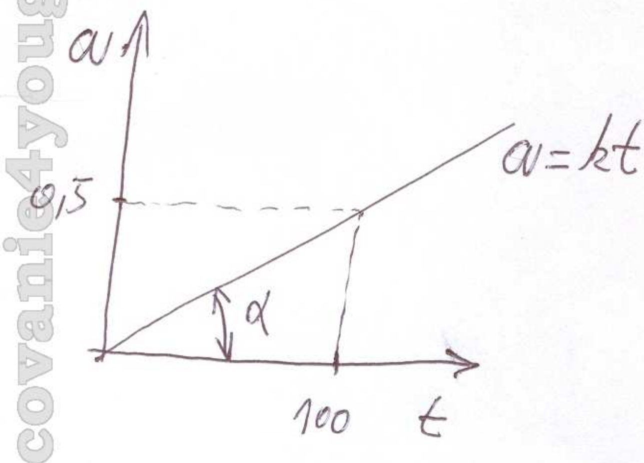
$a_1 = 0{,}5\ m\,s^{-2}$

$v_1 = ?$

$s_1 = ?$



$$k = tg\alpha = \frac{0{,}5}{100} = 5.10^{-3}$$



$$a = kt$$

$$v = a \cdot t$$

$$dv = a\,dt$$

$$dv = (kt)\,dt \quad \Big/ \int$$

$$\int dv = \int kt\,dt$$

$$v = k\frac{t^2}{2} + \begin{matrix} C \\ v_0 \end{matrix}$$

$$\Rightarrow \boxed{v = \frac{1}{2}kt^2 + v_0}$$

$$s = v \cdot t \qquad v = \frac{1}{2}kt^2 + v_0$$

$$ds = v \cdot dt$$

$$ds = \left(\frac{1}{2}kt^2 + v_0\right)dt \quad \int$$

$$\int ds = \int\left(\frac{1}{2}kt^2 + v_0\right)dt$$

$$\boxed{s = \frac{1}{6}kt^3 + v_0 t + \underset{s_0}{C_0}} \qquad v_0 = 0$$

$$v = \frac{1}{2}kt^2 = \frac{1}{2}\,5.10^{-3}.100^2 = 25\ m\,s^{-1}$$

$$s = \frac{1}{6}kt^3 = \frac{1}{6}\,5.10^{-3}.100^3 = 833{,}33\ m$$

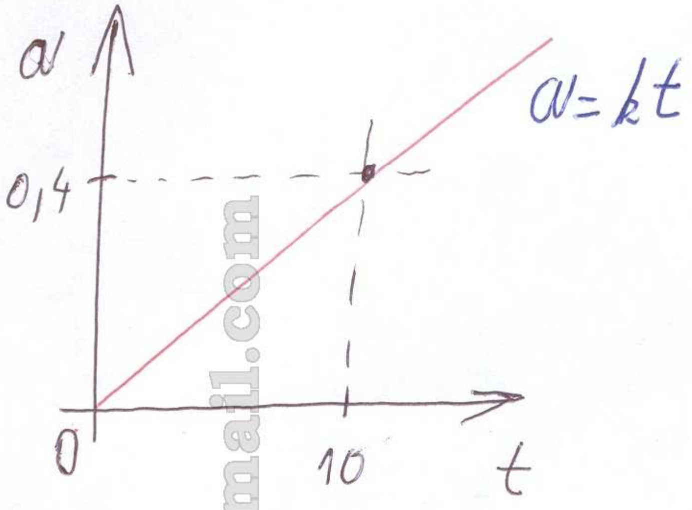
Auto sa rozbieha z pokoja s rovnomerne rastúcim zrýchlením tak, že v $t_1 = 10s$ má hodnotu $a_1 = 0{,}4 m.s^{-2}$ Akú rýchlosť ma auto v čase $t_2 = 20s$ a akú dráhu za ten čas prešlo ?

$t_1 = 10 s$

$a_1 = 0{,}4 m s^{-2}$

$t_2 = 20 s$

$v_2, s_2 = ?$



$k = \frac{0{,}4}{10} = 0{,}04$

$a = \boxed{kt}$

$v = \int a\, dt = \int (kt)\, dt = \boxed{k\frac{t^2}{2} + v_0}$

$s = \int v\, dt = \int \left(k\frac{t^2}{2} + v_0\right) dt = \frac{1}{6}kt^3 + v_0 t + s_0$

$v = \frac{1}{2}kt^2 = \frac{1}{2}0{,}04 . 20^2 = 8 m s^{-1}$

$v_0 = 0$

$s = \frac{1}{6}kt^3 = \frac{1}{6}0{,}04 . 20^3 = 53{,}33 m$

$v_0 = 0 \quad s_0 = 0$

2. Dynamika hmotného bodu

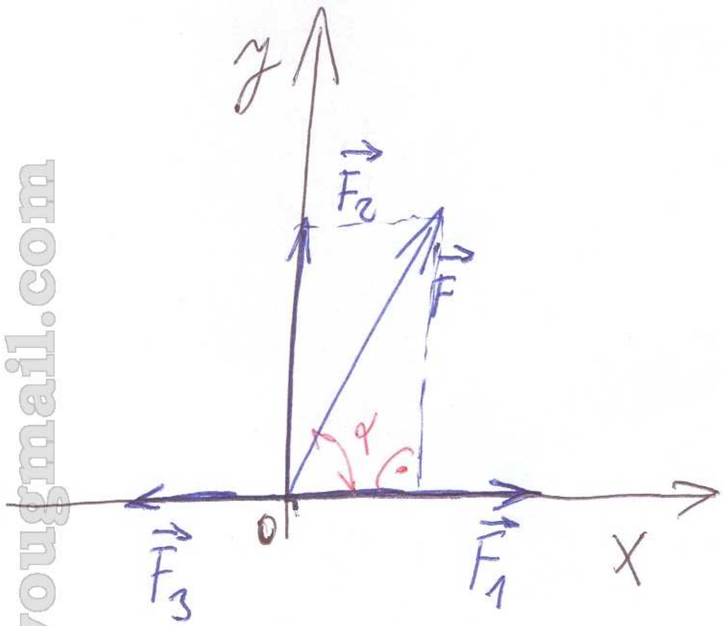
Vypočítajte veľkosť a smer zrýchlenia telesa hmotnosti m = 10kg . Ktoré leží na dokonale hladkej rovine a pôsobia naň tri sily. $F_1 = 6\vec{i}[N]$ $F_2 = 4\vec{j}[N]$ $F_3 = -3\vec{i}[N]$

$m = 10\,kg$

$\vec{F_1} = 6\vec{i}[N] = \vec{F_{x_1}}$

$\vec{F_2} = 4\vec{j}[N] = \vec{F_{y_1}}$

$\vec{F_3} = -3\vec{i}[N] = \vec{F_{x_2}}$



$a = ?$, $\alpha = ?$

$$\vec{F_x} = \sum_{n=1}^{c} \vec{F_{x_n}} = \vec{F_{x_1}} + (-\vec{F_{x_2}}) = 6 + (-3) = 3N$$

$$\vec{F_y} = \sum_{n=1}^{c} \vec{F_{y_n}} = \vec{F_{y_1}} = 4N$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = \sqrt{3^2 + 4^2} = 5N$$

$$\vec{a} = \frac{\vec{F}}{m} = \frac{5}{10} = \underline{0{,}5\,m\,s^{-2}}$$

$$tg\,\alpha = \frac{F_y}{F_x} = \frac{4}{3} \qquad \alpha = arctg\,\frac{4}{3} = \underline{53{,}13^\circ}$$

Častica hmotnosti m sa pohybuje v rovine xy , kde na ňu pôsobí sila $\vec{F}$ kolmá rýchlosť častice $\vec{v}$, pričom v = konštante. Veľkosť sily je priamoúmerná rýchlosti častice v. ukážte , že častica sa pohybuje po kruhovej dráhe a vyjadrite jej polomer ako funkciu rýchlosti.

$$\vec{F} \perp \vec{v}$$

$$\vec{v} = \text{konšt.}$$

$$\vec{F} = k\vec{v}$$

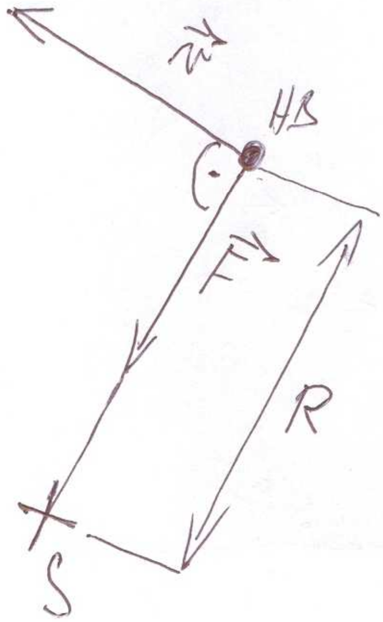
$$\vec{F} = m . \vec{a}$$

$$\vec{v} . k = m\vec{a}$$

$$\cancel{\vec{v}} . k = m \frac{\vec{v}^{2}}{R}$$

$$k = \frac{m\vec{v}}{R}$$

$$\boxed{R = \frac{m\vec{v}}{k}}$$



V silovom poli sa pohybuje teleso hmotnosti m = 5kg po krivke s polohovým vektorom

$\vec{r} = At^3\vec{i} + Bt\vec{j} + C\vec{k}$ kde A = 1 ms^{-3} B = 5ms^{-1} C = -3m Vyjadríte hybnosť a silu ako funkcie času a vypočítajte ich veľkosti v čase t_1 = 2s .

$m = 5\,kg$

$\vec{r} = At^3\vec{i} + Bt\vec{j} + C\vec{k}$

$A = 1\,m\,s^{-3}$

$B = 5\,m\,s^{-1}$

$C = -3\,m$

$t_1 = 2\,s$

$\vec{p} = f(t) \quad \vec{F} = f(t)$

$p_1, F_1 = ?$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(At^3\vec{i} + Bt\vec{j} + C\vec{k}\right) =$$

$$= \frac{d}{dt}\left(At^3\vec{i}\right) + \frac{d}{dt}\left(Bt\vec{j}\right) + \frac{d}{dt}\left(C\vec{k}\right) =$$

$$= 3At^2\vec{i} + B\vec{j} + \vec{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(3At^2\vec{i}\right) + \frac{d}{dt}\left(B\vec{j}\right) + \frac{d}{dt}\left(\vec{k}\right) =$$

$$= 6At\vec{i} = 6.1.2 = 12\,m\,s^{-2}$$

$\vec{F} = m.\vec{a} = 5.12 = \underline{\underline{60N}}$

$$\boxed{\vec{v} = 3At^2\vec{i} + B\vec{j} + \vec{k}}$$

$$\boxed{\vec{a} = 6At\vec{i}}$$

$$\boxed{\vec{F} = m\vec{a} = m(6At\vec{i})}$$

$\vec{p} = m\vec{v} = m(3At^2\vec{i} + B\vec{j} + \vec{k}) =$

$= 5.3.1.2^2\vec{i} + 5.5\vec{j} + 5\vec{k} = 60\vec{i} + 5\vec{j} + 1\vec{k}$

$|\vec{p}| = \sqrt{p_x^2 + p_y^2 + p_z^2} = \sqrt{60^2 + 25^2 + 5^2} = \underline{\underline{65,19 kgms^{-1}}}$

$\vec{p}_x = 60\vec{i}$

$\vec{p}_y = 5\vec{j}$

$\vec{p}_z = 1.\vec{k}$

Teleso hmotnosti $m = 10kg$ sa pohybuje účinkom premennej sily $F = p.(q - t)$, kde $p = 100Ns^{-1}$, $q = 1s$. Za koľko sekúnd sa teleso zastaví, ak v čase $t_0 = 0$ malo rýchlosť $v_0 = 20\ cm\ s^{-1}$ a sila mala smer rýchlosti ? Akú dráhu prejde teleso do zastavenia ?

$m = 10kg$

$F = p(q-t)$

$p = 100 N s^{-1}$

$q = 1s$

$t_0 = 0$

$v_0 = 20 cm s^{-1} = 0{,}2 m s^{-1}$

$\vec{F} \parallel \vec{v}$

$s = ?,\ t_z = ?$

$$\frac{d\vec{p_v}}{dt} = \vec{F}$$

$$d\vec{p_v} = \vec{F}\,dt$$

$$dp_v = p(q-t)\,dt \quad /\int$$

$$\int dp_v = \int p(q-t)\,dt$$

$$\int dp_v = \int pq\,dt - \int pt\,dt$$

$$p_v = p\,g\,t - p\frac{t^2}{2} + \underset{C}{p_0}$$

$$p_0 = v_0 \cdot m = 0{,}2 \cdot 10 = 2$$

$$p_v = p\,g\,t - p\frac{t^2}{2} + 2$$

V ČASE ZASTAVENIA $p_v = 0\ kg\,m\,s^{-1}$

PRETO:

$$0 = p\,g\,t_0 - p\frac{t_0^2}{2} + 2 \qquad t_0 \text{ - ČAS ZASTAVENIA}$$

$$0 = \underset{a}{-50t_0^2} + \underset{b}{100t} + \underset{c}{2} \Big/ \cdot\frac{1}{2} \quad \text{PO DOSADENÍ}$$

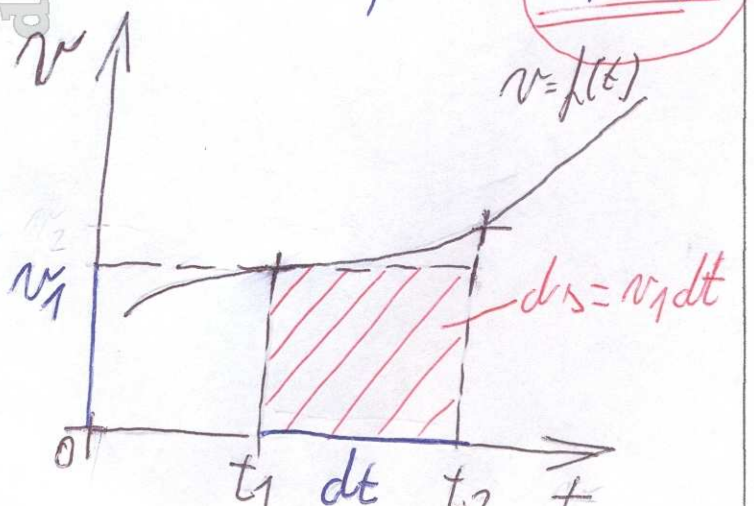
$$0 = \underset{a}{-25t_0^2} + \underset{b}{50t_0} + \underset{c}{1}$$

$$D = b^2 - 4ac = 50^2 - 4\cdot(-25)\cdot 1 = 2600$$

$$t_{12} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-50 \pm 50{,}99}{2\cdot(-25)} = \begin{cases} \cancel{-0{,}0198\,s} \\ 2{,}019\,s \doteq 2{,}02\,s \quad t_0 \end{cases}$$

$$ds = v \cdot dt \Big/ \int$$

$$\int_0^s ds = \int_{t_1}^{t_2} v \cdot dt$$



$$s = \int_{t_1}^{t_2} \boxed{\left(\frac{\mu g t}{m} - \frac{\mu t^2}{2m} + v_0\right)} dt$$

$v \cdot dt$

$$\vec{p_v} = m\vec{v}$$

$$|\vec{v}| = \frac{|\vec{p_v}|}{m} = \frac{1}{m}\left(\mu g t - \frac{t^2}{2}\mu + m v_0\right) =$$

$$= \boxed{\frac{\mu g t}{m} - \frac{\mu t^2}{2m} + v_0} = v$$

$t_1 = 0$

$t_2 = t_0 = 2{,}02\,s$

$$s = \int_{t_1}^{t_2} \left(\frac{\mu g t}{m}\right) dt - \int_{t_1}^{t_2} \left(\frac{\mu t^2}{2m}\right) dt + \int_{t_1}^{t_2} v_0 \, dt =$$

$$= \left[\frac{\mu g}{m}\frac{t^2}{2}\right]_0^{2{,}02} - \left[\frac{\mu}{2m}\frac{t^3}{3}\right]_0^{2{,}02} + \left[v_0 t\right]_0^{2{,}02} =$$

$$= \left[\frac{100 \cdot 1}{10} \cdot \frac{2{,}02^2}{2} - \frac{100 \cdot 1}{10} \cdot \frac{0^2}{2}\right] - \left[\frac{100}{2 \cdot 10} \cdot \frac{2{,}02^3}{3} - \frac{100}{2 \cdot 10} \cdot \frac{0^3}{3}\right] + \left[0{,}2 \cdot 2{,}02 - 0{,}2 \cdot 0\right] = \underline{7{,}074\,m}$$

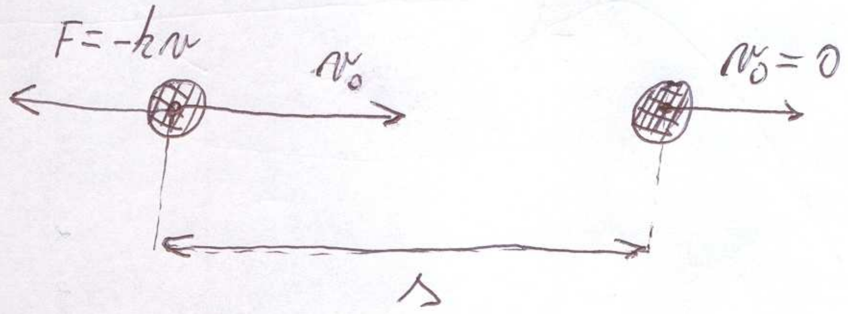
Guľôčka hmotnosti m , ktorá ma začiatočnú rýchlosť v_o , sa pohybuje tak , že odpor prostredia rastie priamoúmerne s rýchlosťou guľôčky. Akú dráhu prejde guľôčka do zastavenia , ak na ňu nepôsobí žiadna sila ?

m

v_0

$F = kv$

$s = ?$



$$m\vec{a} = -\vec{F}$$

$$m\vec{a} = -k\vec{v}$$

$$m\frac{d\vec{v}}{dt} = -k\frac{d\vec{s}}{dt} \Big/ \cdot \frac{1}{dt}$$

$$m\,d\vec{v} = -k\,d\vec{s} \Big/ \int$$

$$\int_{v_0}^{0} m\,dv = -\int_{0}^{s} k\,ds$$

$$\left[mv\right]_{v_0}^{0} = -\left[ks\right]_{0}^{s}$$

$$m\left[0 - v_0\right] = -k\left[s - 0\right]$$

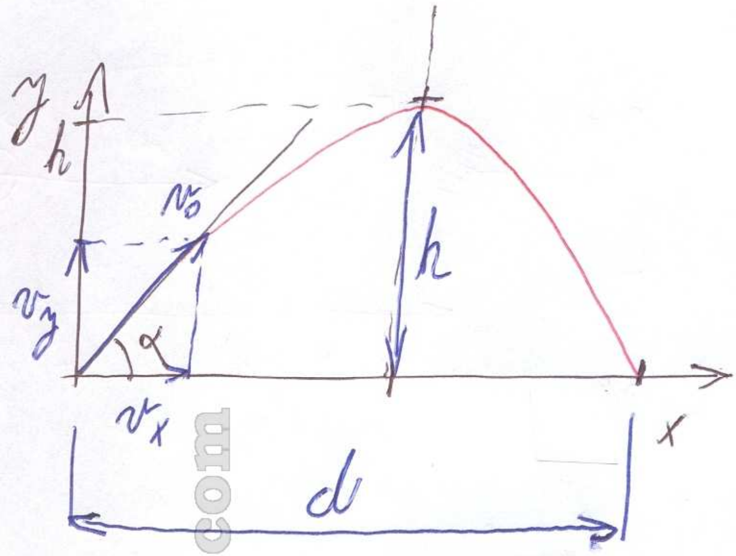
$$-mv_0 = -ks \qquad s = \frac{mv_0}{k}$$

Z požiarnej striekačky tryská voda pod uhlom $\alpha = 60^0$ rýchlosťou $v_o = 50 \text{ m s}^{-1}$. Vypočítajte maximálnu výšku a vzdialenosť miesta ktoré prúd vody dosiahne.

$\alpha = 60^\circ$

$v_0 = 50 \, m s^{-1}$

$h, d = ?$



$v_x = v_0 \cos\alpha$

$v_y = v_0 \sin\alpha$

$x = v_x \cdot t = t v_0 \cos\alpha$

$$y = v_y t - \frac{1}{2} g t^2 = v_0 \sin\alpha - \frac{1}{2} g t^2$$

t_0 - ČAS VÝSTUPU $v_{y_t} = 0$

$v_{y_t} = v_0 \sin\alpha - g t$

$0 = v_0 \sin\alpha - g t_0$

$$t_0 = \frac{v_0 \sin\alpha}{g} = \frac{50 \cdot \sin 60^\circ}{9{,}8} = 4{,}41 \, s$$

DOBA LETU $t_l = 2 t_0 = 2 \cdot 4{,}41 = 8{,}82 \, s$

$$h = \frac{1}{2} g t_0^2 = \frac{1}{2} 9,8 \cdot 4,41^2 = \underline{\underline{95,29 m}}$$

$$d = 2t_0 \cdot v_x = t_1 v_x = t_1 v_0 \cos\alpha =$$

$$= 8,82 \cdot 50 \cos 60° = \underline{\underline{220,5 m}}$$

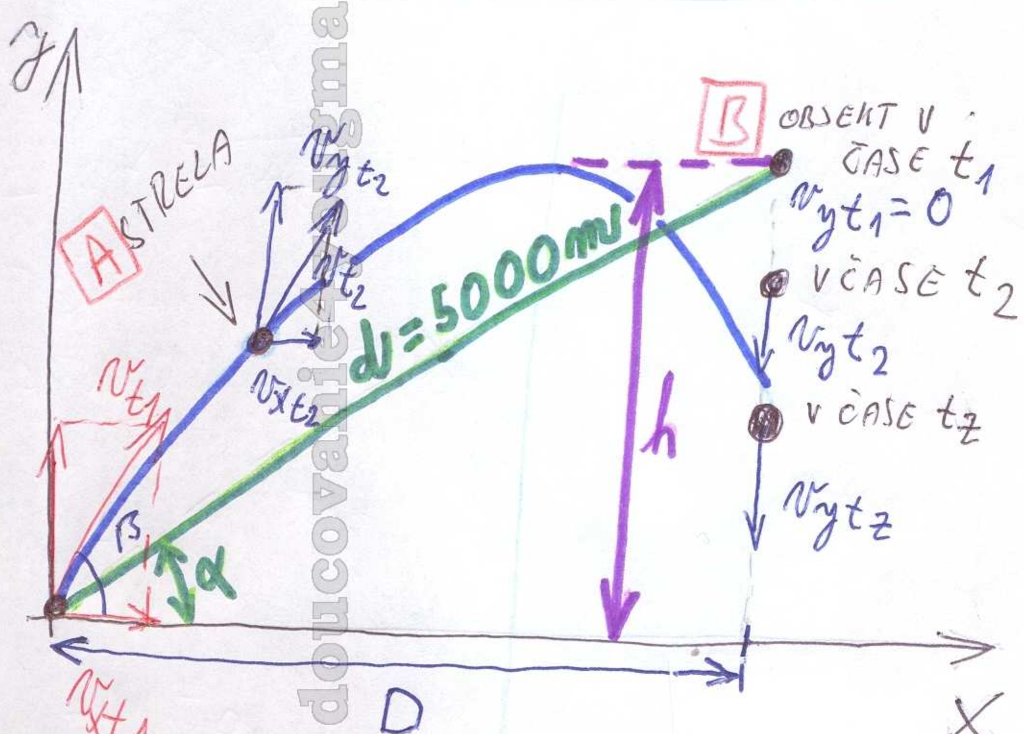
Objekt v priamej vzdušnej vzdialenosti d = 5000m je pozorovaný pod uhlom $\alpha = 60^0$ Aký ma byť výškový uhol výstrelu pri začiatočnej rýchlosti strely $v_o = 300 \text{ m s}^{-1}$, aby bol objekt zasiahnutý , keď začne padať súčastne s výstrelom ? Za aký čas objekt výstrel zasiahne ? Odpor vzduchu zanedbajte.

$d = 5000 m$

$\alpha = 60°$

$v_0 = 300 m s^{-1}$

$\beta, t_z = ?$



$$\cos 60° = \frac{D}{d}$$

$$d \cdot \cos 60° = D = 5000 \cdot \cos 60° = 2500 m$$

$$\sin 60° = \frac{h}{D}$$

$h = d \sin 60° = 5000 \cdot \sin 60° = 4330\ m$

B

$x_B = 2500$

$y_B = h - \frac{1}{2} g t^2$

A

$x_A = \boxed{v_0 \cos \beta} \cdot t$ v_x

$y_A = v_y t - \frac{1}{2} g t^2 = v_0 \sin \beta \cdot t - \frac{1}{2} g t^2$

V ČASE ZRÁŽKY t_z:

$x_B = x_A$ I.

$y_B = y_A$ II.

$2500 = v_0 \cos \beta \cdot t_z$ I.

$h - \cancel{\frac{1}{2} g t_z^2} = v_0 \sin \beta \cdot t - \cancel{\frac{1}{2} g t_z^2}$ II.

$2500 = v_0 \cos \beta \cdot t_z$

$h = v_0 \sin \beta \cdot t_z$

$$\cos^2\beta + \sin^2\beta = 1$$

$$\sin^2\beta = 1 - \cos^2\beta$$

$$\sin\beta = \sqrt{1 - \cos^2\beta}$$

$$2500 = v_0 \cos\beta \cdot t_z$$

$$h = v_0 \sqrt{1 - \cos^2\beta} \cdot t_z$$

$$2500 = v_0 t_z \cos\beta$$

$$\frac{h}{v_0 t_z} = \sqrt{1 - \cos^2\beta} \quad \Big/^2$$

$$2500 = v_0 t_z \cos\beta \Rightarrow \boxed{\cos\beta = \frac{2500}{v_0 t_z}} \; *$$

$$\frac{h^2}{v_0^2 t_z^2} = 1 - \cos^2\beta$$

$$\frac{h^2}{v_0^2 t_z^2} = 1 - \frac{2500^2}{v_0^2 \cdot t_z^2} \Big| \cdot (v_0^2 t_z^2)$$

$$\frac{h^2}{\cancel{v_0^2 t_z^2}} \cdot \cancel{(v_0^2 t_z^2)} = v_0^2 t_z^2 - \frac{2500^2}{\cancel{v_0^2 \cdot t_z^2}} \cdot \cancel{(v_0^2 t_z^2)}$$

$$h^2 = v_0^2 t_z^2 - 2500^2$$

$$\frac{h^2 + 2500^2}{v_0^2} = t_z^2$$

$$t_z = \sqrt{\frac{h^2+2500^2}{v_0^2}} = \sqrt{\frac{4330^2+2500^2}{300^2}} = \underline{\underline{16{,}66\,s}}$$

$$* \cos\beta = \frac{2500}{v_0 t_z}$$

$$\beta = \arccos\frac{2500}{v_0 t_z} = \underline{\underline{60^\circ}}$$

Z dela pobrežného delostrelectva umiestneného vo výške h = 30m nad hladinou mora je vypálená strela pod uhlom $\alpha = 45^0$ vzhľadom na horizontálnu rovinu so začiatočnou rýchlosťou $v_o = 1000\ m\ s^{-1}$. Aká je vodorovná vzdialenosť miesta v ktorom strela zasiahne cieľ ležiaci na hladine mora, od dela ?

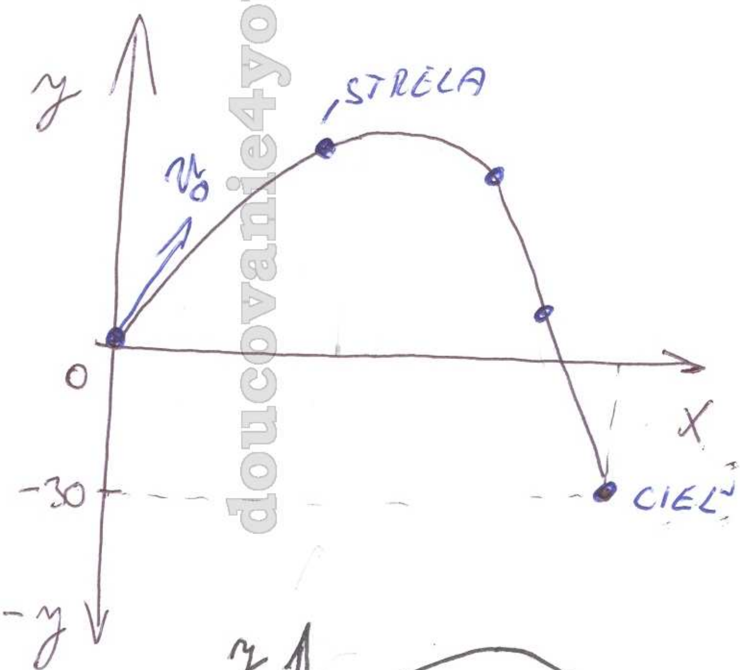
$h = 30\,m$

$v_0 = 1000\,m\,s^{-1}$

$\alpha = 45^\circ$

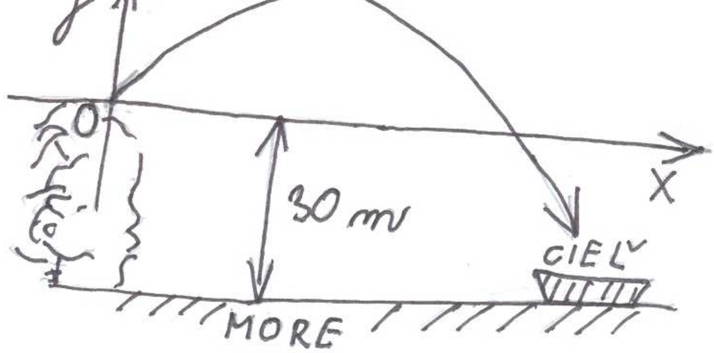
$v_x = v_0 \cos\alpha$

$v_y = v_0 \sin\alpha$



$$y = v_y t - \frac{1}{2} g t^2$$

$$x = v_x t$$



$y = -3 \quad x = d$

$$-30 = v_0 \sin\alpha \, t_z - \frac{1}{2} g t_z^2$$

$$d = v_0 \cos\alpha \, t_z$$

$$\frac{1}{2} g t_z^2 - \sin\alpha \, t_z - 30 = 0$$

a b c

$$D = b^2 - 4ac = (v_0 \sin\alpha)^2 - 4 \cdot \frac{1}{2} g \cdot (-30) = 500058{,}8$$

$$\sqrt{D} = 707{,}15$$

$$t_{z_{1,2}} = \frac{-b \pm \sqrt{D}}{2a} = \frac{v_0 \sin\alpha \pm 707{,}15}{g} =$$

$$= \frac{707{,}1 \pm 707{,}15}{9{,}8} = \begin{cases} 144{,}31\,s = t_z \\ \cancel{-5{,}1 \cdot 10^{-3}\,s} \end{cases}$$

$$x = d = v_x t_z = v_0 \cos\alpha \cdot t_z = 1000 \cos 45° \cdot 144{,}31\,s$$

$$x = d = 101{,}8 \cdot 10^3\,m = 101{,}8\,km$$

Určte polohu ťažiska drôtu ohnutého do tvaru štvrťkružnice s polomerom R = 10 cm

$R = 10\ cm$

$x_T, y_T = ?$

$\frac{X}{R_1} = \cos\alpha$

$R_1 = R_2 = R$

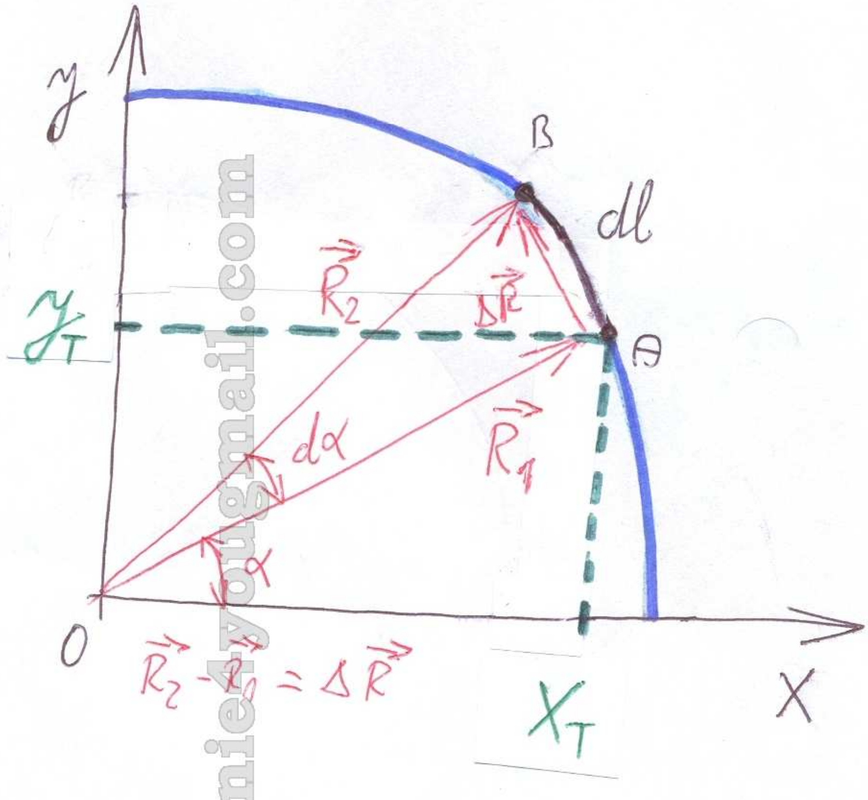
$X = R\cos\alpha$

$\frac{dm}{dl} = \frac{m}{l} = \rho$

$dm = \frac{m}{l}\,dl = \rho\,dl$

$dl = d\alpha \cdot R$

$r_T = \frac{1}{m}\int_T r \cdot dm$

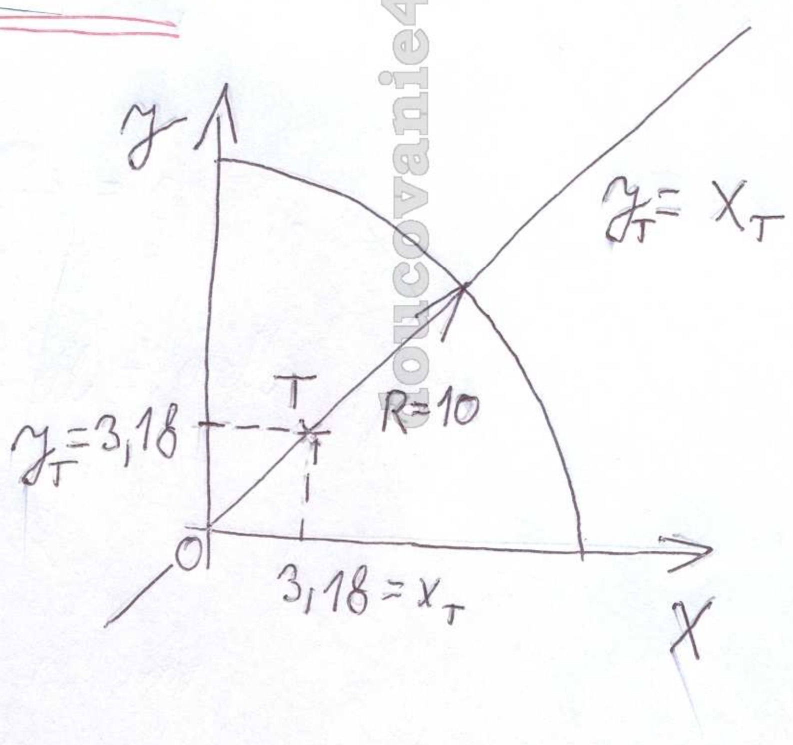


$$x_T = \frac{1}{m}\int_l x\,dm = \frac{1}{m}\int_0^{\pi/2} \underbrace{\boxed{R\cos\alpha}}_{x}\ \cdot\ \underbrace{\boxed{\frac{m}{l}\underbrace{R\,d\alpha}_{dl}}}_{dm} =$$

$$= \frac{R^2}{l}\int_0^{\pi/2}\cos\alpha\,d\alpha = \frac{R^2}{\frac{2\pi R}{4}}\Big[\sin\alpha\Big]_0^{\pi/2} =$$

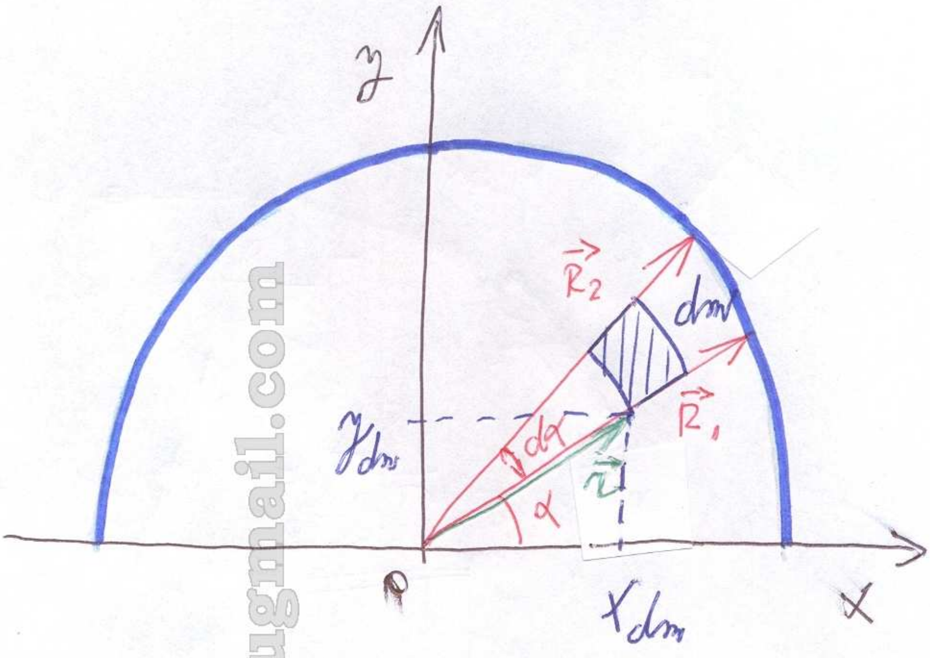
$$= \frac{4R}{2\pi}\left[\sin\frac{\pi}{2} - \sin 0\right] = \frac{2R}{\pi}$$

$$x_T = y_T = \frac{2R}{\pi} = \frac{10.2}{6{,}28} = 3{,}18\,cm$$

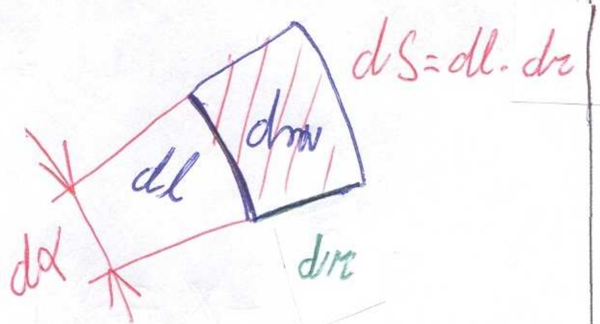


Určte polohu ťažiska homogénneho telesa, ktoré ma tvar polkruhovej dosky zanedbateľnej hrúbky s polomerom R.

$$\frac{R}{x_T, y_T = ?}$$



$$dl = r \cdot d\alpha$$



$$\frac{m}{S} = \frac{dm}{dS}$$

$$dm = \frac{m}{S} \cdot dS = \frac{m}{\frac{\pi R^2}{2}} dl \cdot dr =$$

$$= \frac{2m}{\pi R^2} r \cdot d\alpha \cdot dr$$

$$\frac{x}{r} = \cos\alpha \qquad x = r\cos\alpha$$

$$x \to x_{dm}$$

$$x_T = \frac{1}{m} \int_T x \, dm$$

$$X_T = \frac{1}{m}\int_0^R\int_0^\pi \boxed{r\cdot\cos\alpha}\cdot\boxed{\frac{2m}{\pi R^2}\, r\, d\alpha\cdot dr} =$$

$$X \cdot dm$$

$$\int_S$$

$$= \frac{2}{\pi R^2}\int_0^R\int_0^\pi r^2\cos\alpha\, d\alpha\cdot dr = \frac{2}{\pi R^2}\int_0^\pi\left[\frac{r^3}{3}\right]_0^R \cos\alpha\, d\alpha =$$

$$= \frac{2}{\pi R^2}\int_0^\pi \frac{R^3}{3}\cos\alpha\, d\alpha = \frac{2R}{3\pi}\left[\sin\alpha\right]_0^\pi =$$

$$= \frac{2R}{3\pi}\cdot 0 = \underline{0 = X_T}$$

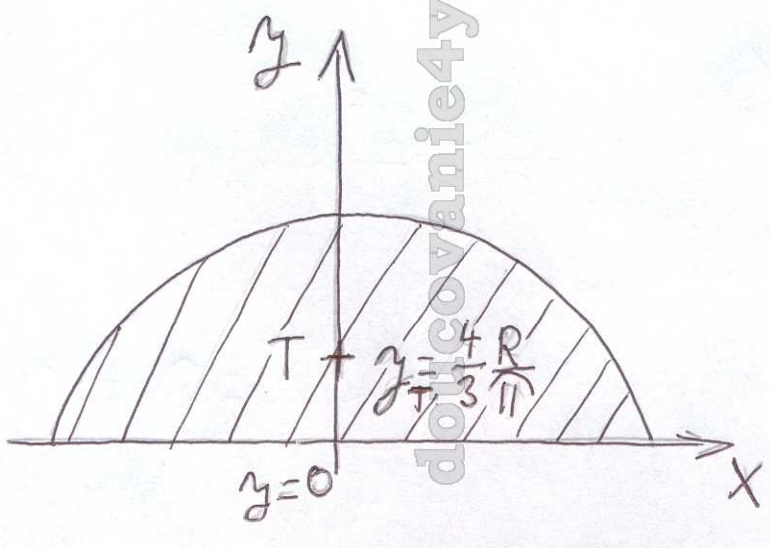
$$y_T = \frac{1}{m}\int_S y\cdot dm = \frac{1}{m}\int_0^R\int_0^\pi \boxed{r\cdot\sin\alpha}\,\boxed{\frac{2m}{\pi R^2}\cdot r\, d\alpha\, dr} =$$

$$y \cdot dm$$

$$= \frac{2}{\pi R^2}\int_0^{\pi}\left[\frac{r^3}{3}\right]_0^R \sin\alpha \, d\alpha = \frac{2}{\pi R^2}\int_0^{\pi}\frac{R^3}{3}\sin\alpha \, d\alpha =$$

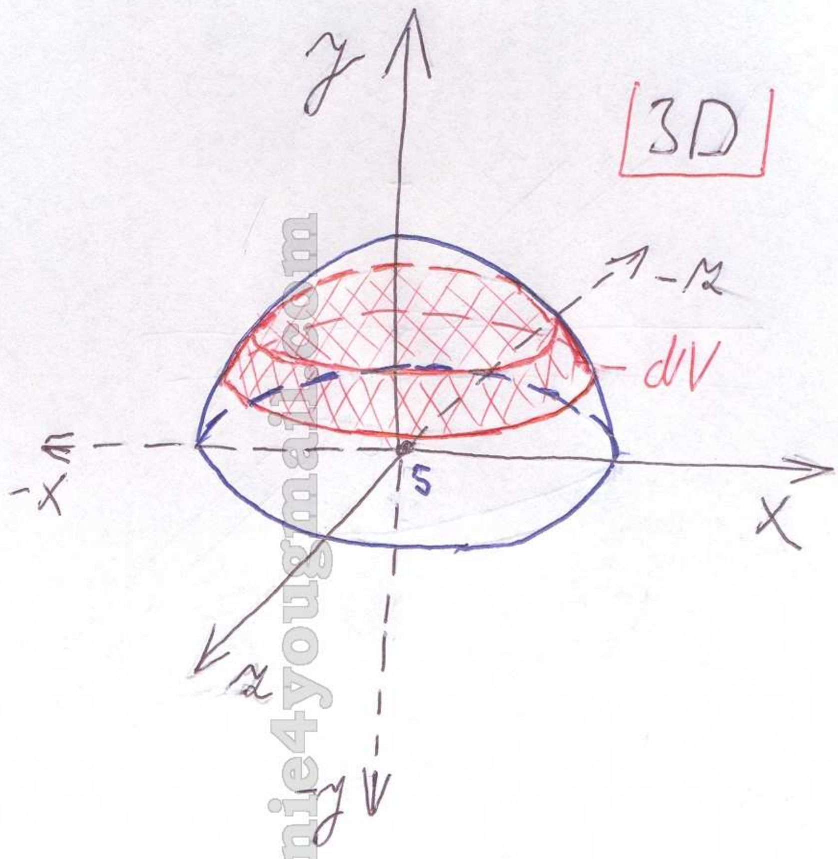
$$= -\frac{2R}{3\pi}\int_0^{\pi} -\sin\alpha \, d\alpha = -\frac{2R}{3\pi}\left[\cos\alpha\right]_0^{\pi} =$$

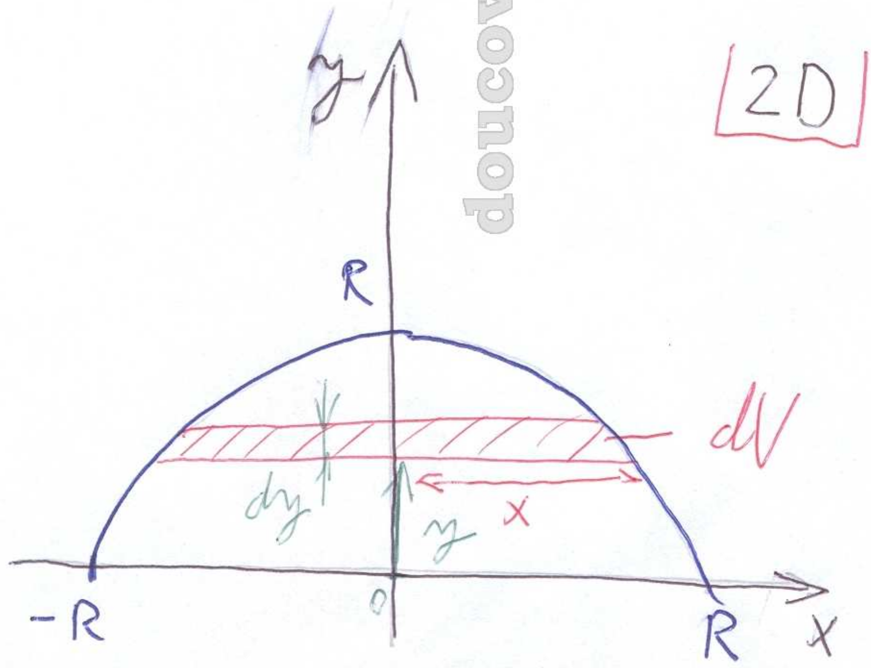
$$= -\frac{2R}{3\pi}\left[-1-1\right] = \frac{4R}{3\pi} = y_T$$



Určte polohu ťažiska homogénneho telesa tvaru polgule s polomerom R.

$$\frac{R}{x_T, y_T, z_T = ?}$$





$$x^2 + y^2 = R^2$$

$$x = \sqrt{R^2 - y^2}$$

$$dV = \underbrace{\boxed{\pi x^2}}_{dS} \cdot dy = \pi\left(\sqrt{R^2 - y^2}\right)^2 dy$$

$$dm = \rho\, dV = \underbrace{\frac{m}{V}}_{\rho} \cdot dV = \frac{m}{\frac{\frac{4}{3}\pi R^3}{2}}\, dV =$$

$$= \frac{3m}{2\pi R^3}\, dV = \underbrace{\boxed{\frac{3m}{2\pi R^3}}}_{\rho} \cdot \underbrace{\boxed{\pi(R^2 - y^2) \cdot dy}}_{dV} = dm$$

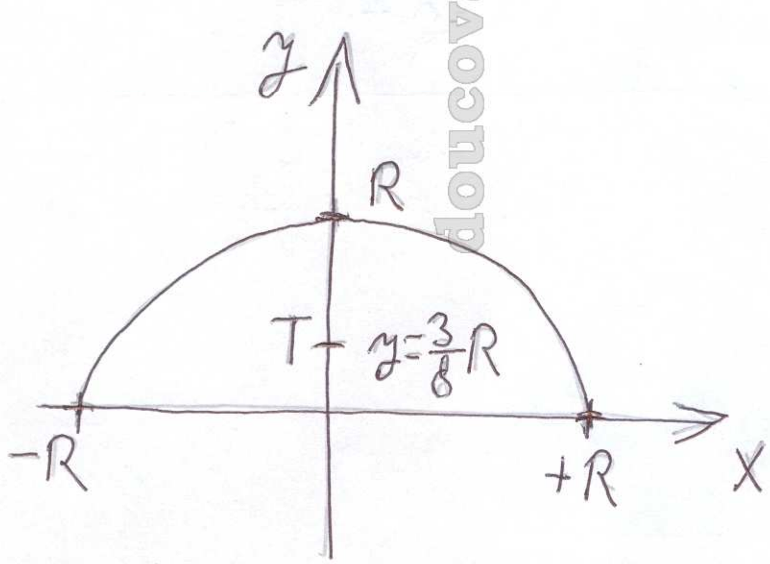
$$y_T = \frac{1}{m}\int_T y\, dm = \frac{1}{m}\int_0^R \underbrace{\boxed{y}}_{y} \cdot \underbrace{\boxed{\frac{3m}{2\pi R^3}\pi(R^2 - y^2)\, dy}}_{dm} =$$

$$= \frac{3}{2R^3} \int_0^R y\left(R^2 - y^2\right) dy = \frac{3}{2R^3}\left[R^2 \frac{y^2}{2} - \frac{y^4}{4}\right]_0^R =$$

$$= \frac{3}{2R^3}\left[\left(R^2 \frac{R^2}{2} - \frac{R^4}{4}\right) - \left(R^2 \frac{0^2}{2} - \frac{0^4}{4}\right)\right] =$$

$$= \frac{3}{2R^3}\left(\frac{R^4}{4}\right) = \frac{3}{8} R = y_T$$

$$X_T = z_T = 0$$



Vozík s pieskom hmotnosti m_1= 100 kg sa pohybuje rovnomerným priamočiarym pohybom po vodorovnej rovine rýchlosťou $v_1 = 1\ ms^{-1}$. Oproti nemu letí guľa hmotnosti $m_2 = 2$ kg rýchlosťou $v_2 = 70\ ms^{-1}$, narazí na vozík a zaryje sa do piesku. Ako sa bude vozík pohybovať po dopade gule ?

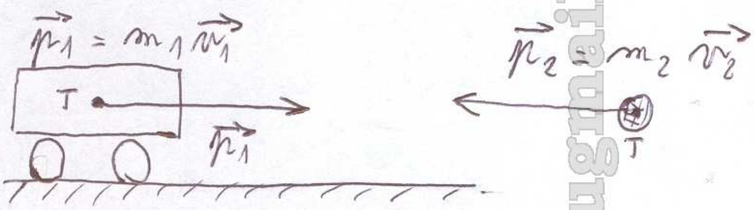
$v_1 = 1\,m\,s^{-1}$

$m_1 = 100\,kg$

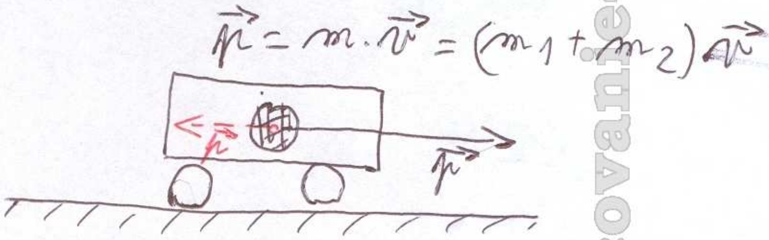
$v_2 = 70\,m\,s^{-1}$

$m_2 = 2\,kg$

$v = ?$



I.



II.

$$\vec{p}_1 + (-\vec{p}_2) = \vec{p}$$

$$m_1 v_1 + (-m_2 v_2) = (m_1 + m_2)\, v$$

$$\frac{m_1 v_1 - m_2 v_2}{m_1 + m_2} = v = \frac{100 \cdot 1 - 2 \cdot 70}{100 + 2} = \underline{\underline{-0{,}39\,m\,s^{-1}}}$$

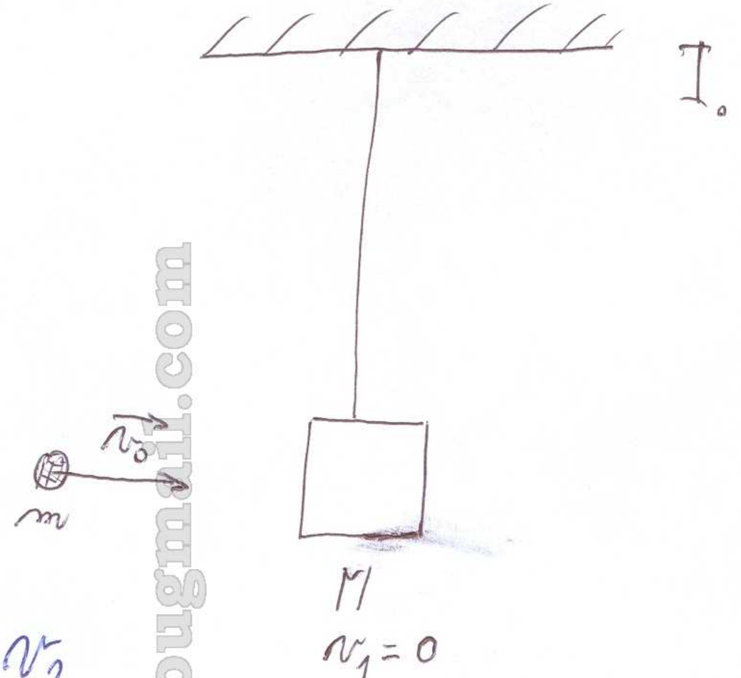
Do telesa hmotnosti M zaveseného na tenkej niti narazí guľôčka hmotnosti m letiaca rýchlosťou v_o v horizontálnom smere a uviazne v ňom. Do akej výšky h po vychýlení z rovnovážnej polohy vystúpi teleso po náraze guľôčky ?

M

m

v_0

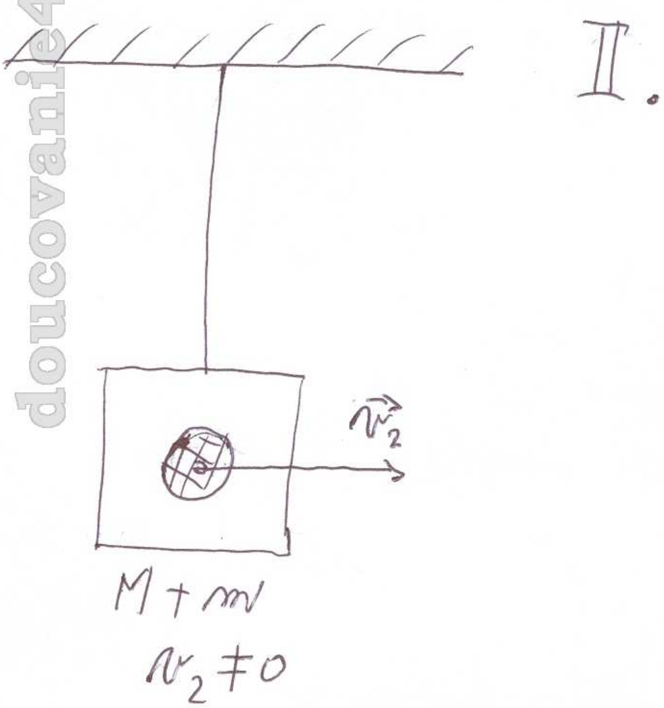
$h = ?$



$$m v_0 + M v_1 = (M + m) v_2$$

$v_1 = 0$

$$\frac{m v_0 + M v_1}{M + m} = v_2$$

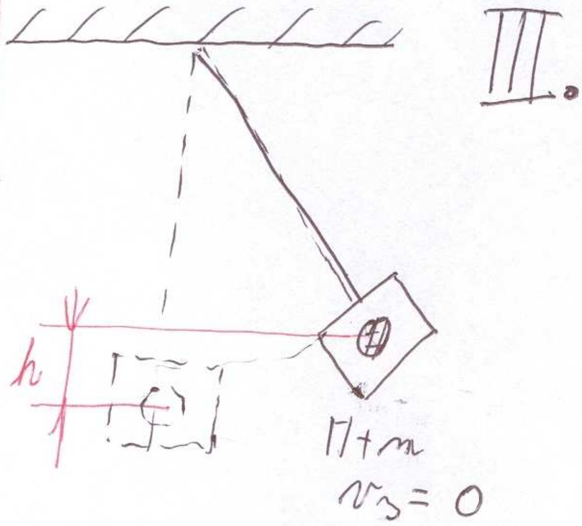


$$\frac{m v_0}{M + m} = v_2$$

$$E_p = E_K$$

$$\boxed{hg(M+m)} = \boxed{\frac{1}{2}(M+m)\frac{m^2 v_0^2}{(M+m)^2}}$$

$$E_p = E_k$$



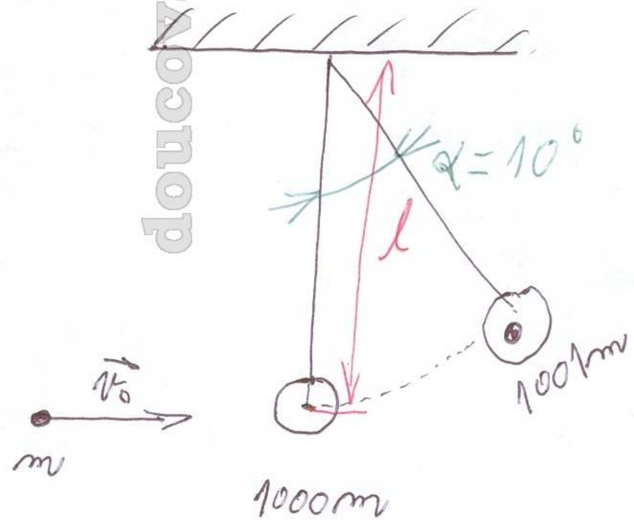
$$hg(M+m) = \frac{1}{2}\frac{(mv_0)^2}{(M+m)}$$

$$\boxed{h = \frac{(mv_0)^2}{2g(M+m)^2}}$$

Do telesa tvaru gule zaveseného na tenkom vlákne, narazí vodorovne letiaca strela, ktorej hmotnosť je 1000 krát menšia ako hmotnosť telesa a uviazne v ňom. Aká bola rýchlosť strely pri náraze, keď sa teleso po náraze vychýlilo zo svojej rovnovážnej polohy, tak že záves zvieral zo zvislým smerom uhol 10^0 ? Dĺžka závesu (po stred gule) je l =1 m.

l
m
$M = 1000m$
$\alpha = 10^0$

$v_0 = ?$



$$\frac{(l-h)}{l} = \cos\alpha$$

$$(l-h) = l\cos\alpha$$

$$h = l(1-\cos\alpha)$$

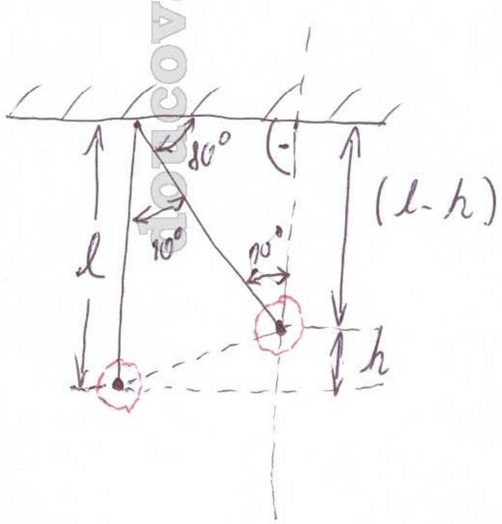
$$h = \frac{m^2 v_0^2}{2g(M+m)^2}$$

$$l(1-\cos\alpha) = \frac{m^2 v_0^2}{2g(1001m)^2}$$

$$v_0 = \sqrt{\frac{l(1-\cos\alpha)\,2g(1001)^2 \cancel{m^2}}{\cancel{m^2}}} =$$

$$= \sqrt{l(1-\cos\alpha)\,2g} \cdot 1001 = \sqrt{1 \cdot (1-\cos 10^\circ) \cdot 2 \cdot 9{,}81} \cdot 1001$$

$$\underline{\underline{v_0 = 546{,}22\ m\,s^{-1}}}$$



Učte moment zotrvačnosti homogénnej tyče dĺžky l a hmotnosti m vzhľadom na os kolmú na tyč prechádzajúcu.

a, koncovým bodom tyče

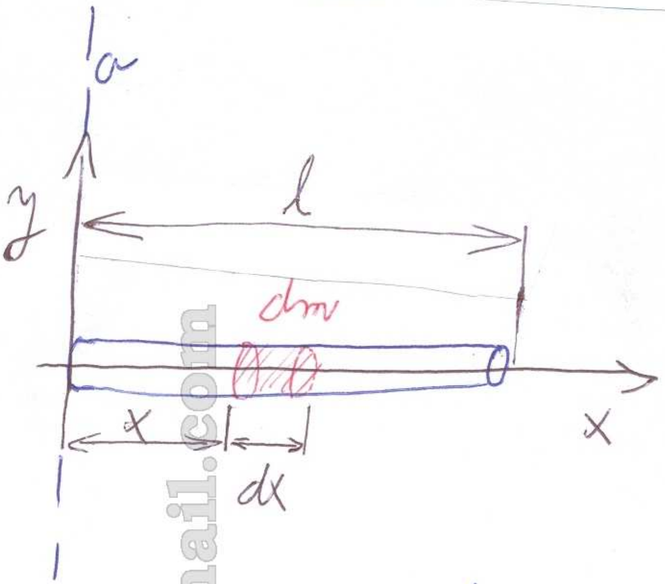
b, stredom tyče.

I.

l

m

$J = ?$



$J = m r^2$

$$\boxed{J = \sum_{i=1}^{n} m_i \cdot x_i^2}$$

$$\frac{m}{l} = \frac{dm}{dx} = \rho$$

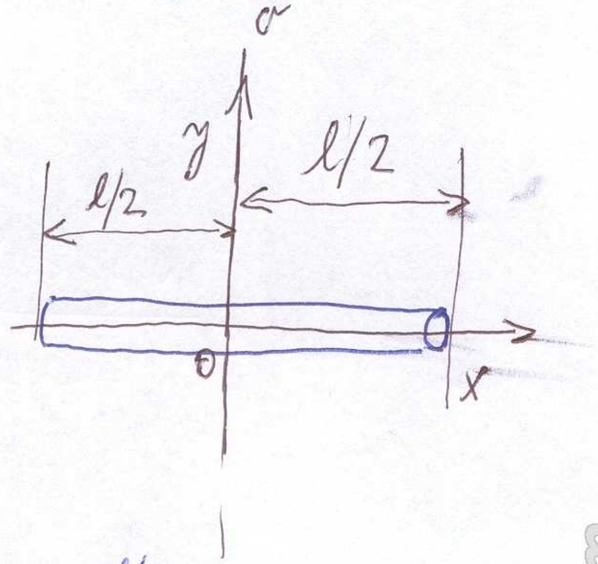
$$dm = \frac{m}{l}\, dx$$

$$dJ = dm \cdot x^2 \Big/ \int$$

$$\int_T dJ = \int_T x^2\, dm$$

$$J = \int_0^l x^2\, dm = \int_0^l x^2 \frac{m}{l}\, dx = \frac{m}{l}\left[\frac{x^3}{3}\right]_0^l =$$

$$= \frac{m}{l}\frac{l^3}{3} = \frac{1}{3} m l^2 = J$$



II.

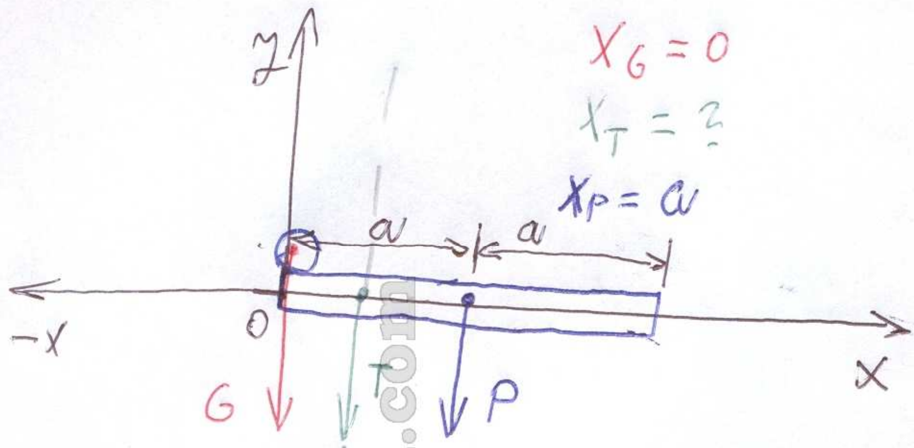
$$J = 2\int_0^{l/2} x^2 \frac{m}{l}\, dx = 2\frac{m}{l}\left[\frac{x^3}{3}\right]_0^{l/2} =$$

$$= \frac{2m}{l}\left(\frac{\frac{l^3}{8}}{3}\right) = \frac{2m}{l}\frac{l^3}{24} = \frac{1}{12} m l^2 = J$$

Na jednom konci loďky, ktorá pokojne stojí na vode, je človek. O koľko sa loďka posunie ,ak človek prejde na jej druhý koniec, keď tiaž človeka je G, tiaž loďky P a dĺžka loďky 2a. Odpor vody pri pohybe loďky zanedbajte.

$G, P, s = 2a$

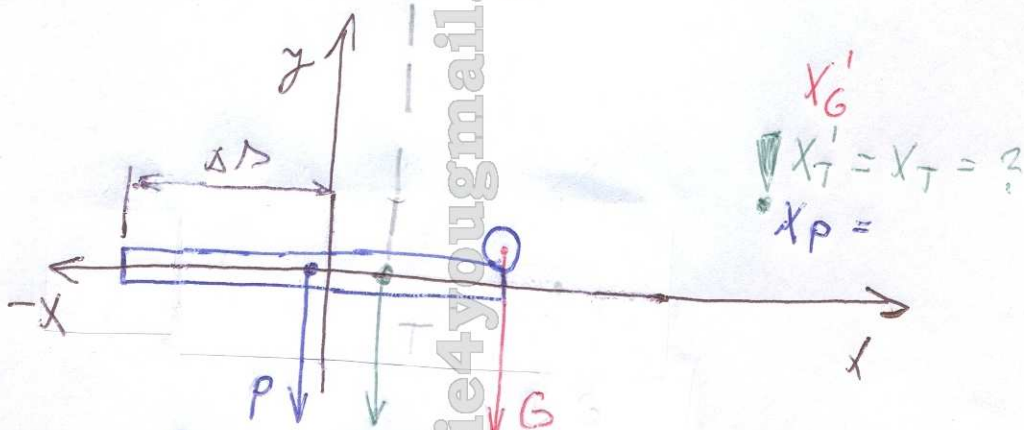
$\Delta s = ?$



$X_G = 0$

$X_T = ?$

$X_P = a$



X_G'

$X_T' = X_T = ?$

$X_P =$

$$X_T = \frac{X_G \frac{G}{m} + X_P \frac{P}{m}}{\frac{G+P}{m}} = \frac{\underset{0}{X_G} \cdot G + X_P P}{G+P} = \frac{X_P \cdot P}{G+P}$$

$$X_T' = \frac{X_G' \frac{G}{m} + X_P' \frac{P}{m}}{\frac{G+P}{m}} = \frac{X_G' \cdot G + X_P' P}{G+P}$$

$$X_T = X_T' \qquad X_G' = (2a - \Delta s) \qquad X_P' = (a - \Delta s)$$

$$X_P = \frac{2a}{2} = a \qquad X_G = 0$$

$$x_T' = \frac{(2a-\Delta s).G + (a-\Delta s).P}{G+P} = x_T$$

$$\frac{(2a-\Delta s).G + (a-\Delta s).P}{\cancel{G+P}} = \frac{x_P.P}{\cancel{G+P}}$$

$$(2a-\Delta s).G + (a-\Delta s)P = x_P . P \qquad x_P = a$$

$$2aG - \Delta sG + \cancel{Pa} - \Delta sP = \cancel{a.P}$$

$$-\Delta sG - \Delta sP = -2aG \;/\cdot(-1)$$

$$\Delta s(G+P) = 2aG$$

$$\Delta s = \frac{2aG}{G+P}$$

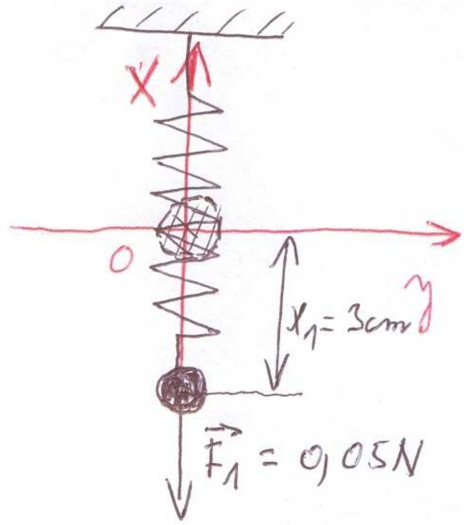
4. Mechanické kmity a vlny

Vypočítajte dobu kmitu T netlmeného harmonického kmitavého pohybu častice hmotnosti m = 10g , ak sila udržujúca časticu v tomto pohybe ma pri výchylke X_1= 3 cm hodnotu $F_1 = 0{,}05$ N

$m = 10\,g = 0{,}01\,kg$

$x_1 = 3\,cm = 0{,}03\,m$

$F_1 = 0{,}05\,N$



$$\omega = \sqrt{\frac{k}{m}}$$

$$F = k \cdot x \qquad k = \frac{F}{x}$$

$$T = \frac{2\pi}{\omega}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\frac{F}{x}}{m}} = \sqrt{\frac{F}{x\,m}}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{F}{x\,m}}} = \sqrt{\frac{4\pi^2}{\frac{F}{x\,m}}} = \sqrt{\frac{4\pi^2 x\,m}{F}} =$$

$$= \sqrt{\frac{4\pi^2 \cdot 0{,}03 \cdot 0{,}01}{0{,}05}} = \underline{\underline{0{,}486\,s}}$$

Doba kmitu netlmeného harmonického kmitavého pohybu je T. Určite v akom čase bude veľkosť rýchlosti bodu rovná polovici jeho maximálnej rýchlosti. Riešte úlohu pri počiatočných podmienkach pre t = 0 , výchylka z rovnovážnej polohy x = A a rýchlosť v = 0

T

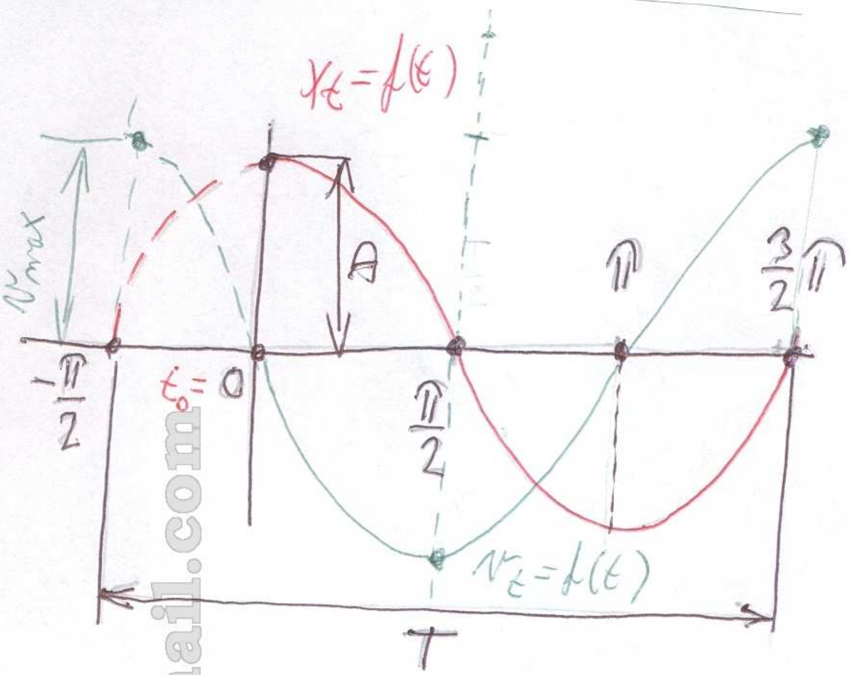
$v_{t_1} = \frac{v_{max}}{2}$

$x = A$

$v = 0$

$t_0 = 0$

$t_1 = ?$



$x_t = A\cos(\omega t)$

$x_t = A\sin\left(\omega t \underbrace{-\frac{\pi}{\pi}}_{\alpha}\right)$

$x_t = A\cos\omega t$

$v_t = \frac{ds}{dt} = \frac{dx_t}{dt} = \frac{d}{dt}\left(A\cos(\omega t)\right)$

$v_t = -A\sin(\omega t) \cdot \omega = \underbrace{-A\omega}_{v_{max}}\sin(\omega t) = v_t$

$v_t = -A\omega\sin(\omega t)$

$v_{t_1} = \frac{v_{max}}{2} = \frac{\cancel{-A\omega}}{2} = \cancel{-A\omega}\sin(\omega t_1)$

$$\frac{1}{2} = \sin(\omega t_1) \quad / \arcsin$$

$$\arcsin\frac{1}{2} = \arcsin \sin(\omega t_1)$$

$$\frac{\pi}{6} = \omega t_1$$

$$T = \frac{2\pi}{\omega}$$

$$\omega = \frac{2\pi}{T}$$

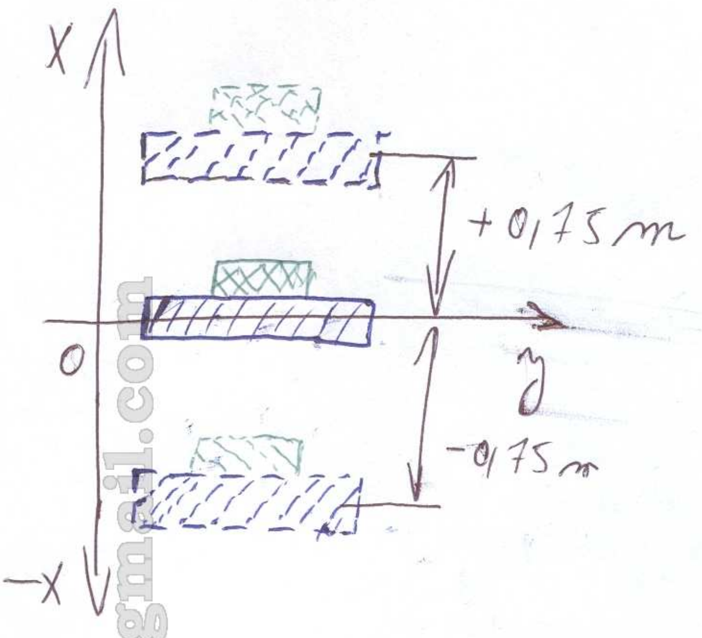
$$\frac{\pi}{6} = \frac{2\pi}{T} t_1$$

$$\frac{\frac{\cancel{\pi}}{6}}{\frac{2\cancel{\pi}}{T}} = \boxed{t_1 = \frac{T}{12}}$$

Kruhová doska uložená vodorovne koná v zvislom smere kmitavý pohyb s amplitúdou 0,75 m. Aká môže byť maximálna frekvencia kmitania dosky, aby predmet voľne uložený na doske sa od nej neoddelil ?

$A = \pm 0{,}75\, m$

$f = ?$



$x = A \sin(\omega t)$

$T = \frac{2\pi}{\omega}$

$f = \frac{1}{T} = \frac{\omega}{2\pi}$

$\omega = 2\pi f$

$v = \frac{dx}{dt} = \frac{d}{dt}\left(A \sin(\omega t)\right) = \underbrace{A\omega}_{v_{max}} \cos(\omega t)$

$a = \frac{dv}{dt} = \frac{d}{dt}\left(A\omega \cos(\omega t)\right) = \underbrace{-A\omega^2}_{a_{max}} \sin(\omega t)$

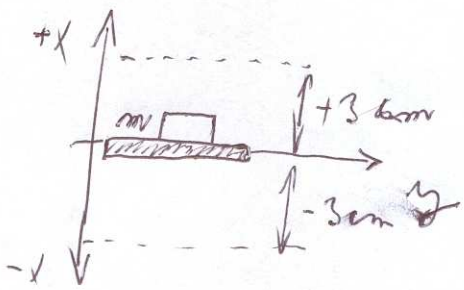
$a_{max} = g$!

$-A\omega^2 = g$

$-A(2\pi f)^2 = g$

$f = \sqrt{\frac{g}{-A 4\pi^2}} = \sqrt{\frac{9{,}81}{-(-0{,}75) \cdot 4\pi^2}} = \underline{\underline{0{,}575\, Hz}}$

Na doske leží závažie hmotnosti m = 2kg . Doska koná harmonický pohyb vo zvislom smere s dobou kmitu T = 0,5s a amplitúdou A = 3cm. Vyjadrite silu, ktorou závažie tlačí na dosku a vypočítajte maximálnu hodnotu tejto sily.



$m = 2\,kg$

$T = 0{,}5\,s$

$A = 3\,cm = 0{,}03\,m$

$F = f(t)$

$F_{max} = ?$

$X = A \sin(\omega t)$

$$a = \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}\left(A \sin(\omega t)\right) = -\omega^2 A \sin(\omega t)$$

$a_{max} = -\omega^2 A \qquad \omega = \frac{2\pi}{T}$

$$F_{max} = a_{max} \cdot m = -\omega^2 A \cdot m = -\left(\frac{2\pi}{T}\right)^2 \cdot A \cdot m$$

$$F_{max} = -\frac{4\pi^2}{0{,}5^2} \cdot 0{,}03 \cdot 2 = -9{,}47\,N$$

!?

$a' = 0 \qquad \left[a = f(t)\right]' = 0$

$\left[-\omega^2 A \sin(\omega t_0)\right]' = 0$

$$-\omega^2 A\cos(\omega t_0)\cdot\omega = 0 \quad \Big/ \cdot \frac{1}{-\omega^3 A}$$

$$\cos(\omega t_0) = 0 \qquad \omega t_0 = \varphi$$

$$\cos\varphi = 0 \quad \Big/ \arccos$$

$$\arccos\cos\varphi = \arccos 0$$

I. $\varphi = \frac{\pi}{2}$ $\qquad \omega = \frac{2\pi}{T} = 4\pi = 12{,}56$

$$\omega t_{0_1} = \frac{\pi}{2}$$

$$t_{0_1} = \frac{\pi}{2}\cdot\frac{1}{\omega} = \frac{\cancel{\pi}}{2}\cdot\frac{T}{2\cancel{\pi}} = \frac{T}{4} = \frac{0{,}5}{4} = \underline{\underline{0{,}125\,s}}$$

V ČASE t_{0_1} NASTÁVA EXTRÉM

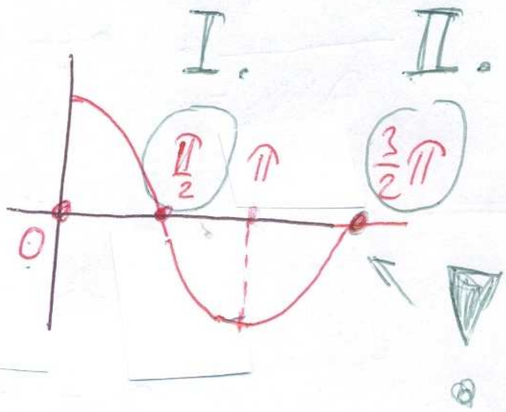
$$F_1 = m\cdot a = m\cdot\left(-\omega^2 A\sin(\omega t_{0_1})\right) =$$

$$= -2\cdot\left(12{,}56^2\cdot 0{,}03\sin(12{,}56\cdot 0{,}125)\right) =$$

$$= -9{,}46\,\underbrace{\sin(1{,}57)}_{=1} = \underline{\underline{-9{,}46\,N}}$$

ALE:

$\cos(\omega t_0) = 0$



II.

$\omega t_{0_2} = \frac{3}{2}\pi \quad \left(\frac{\pi}{2}\right)$

$t_{0_2} = \frac{3}{2}\pi \cdot \frac{1}{\omega} = \frac{3}{2}\pi \cdot \frac{T}{2\pi} = \frac{3}{4} \cdot T = \frac{3}{4} \cdot 0{,}5 = \underline{\underline{0{,}375\,s}}$

$F_2 = -m\left(\omega^2 A \sin(\omega t_{01})\right) =$

$= -2 \cdot (12{,}56^2 \cdot 0{,}03 \sin(12{,}56 \cdot 0{,}375)$

$= -9{,}46 \cdot \sin 4{,}71 = \underline{\underline{9{,}46\,N}}$

$\sin 4{,}71 = -1$

$\boxed{\begin{array}{l} F_1 = F_{min} \\ F_2 = F_{max} \end{array}}$

! $|F_{min}| = F_{max}$

Určite amplitúdu a fázovú konštantu netlmeného harmonického pohybu častice ak, v čase $t_0 = 0$ bola výchylka $X_1 = 5\,cm$ a rýchlosť $v_1 = 20\,cms^{-1}$, frekvencia pohybu je $1\,s^{-1}$

$t_0 = 0$

$X_1 = 5\,cm$

$v_1 = 20\,cm\,s^{-1}$

$f = 1\,Hz$

$X_0, \alpha = ?$

$$X_t = X_0 \sin(\omega t + \alpha) \quad \mathrm{I.}$$

$$v = \frac{dX_t}{dt} = \frac{d}{dt}\left(X_0 \sin(\omega t + \alpha)\right) = \omega X_0 \cos(\omega t + \alpha)$$

$$5 = X_0 \sin(\omega \cdot 0 + \alpha)$$

$$5 = X_0 \sin\alpha \quad \mathrm{I.}$$

$$\omega = 2\pi f$$

$$v = \omega X_0 \cos(\omega t + \alpha) \quad \mathrm{II.}$$

$$20 = 2\pi \cdot f X_0 \cos(2\pi f \cdot 0 + \alpha)$$

$$20 = 2\pi X_0 \cos\alpha \quad \mathrm{II.}$$

$$5 = X_0 \sin\alpha \quad \mathrm{I.} \Rightarrow X_0 = \frac{5}{\sin\alpha}$$

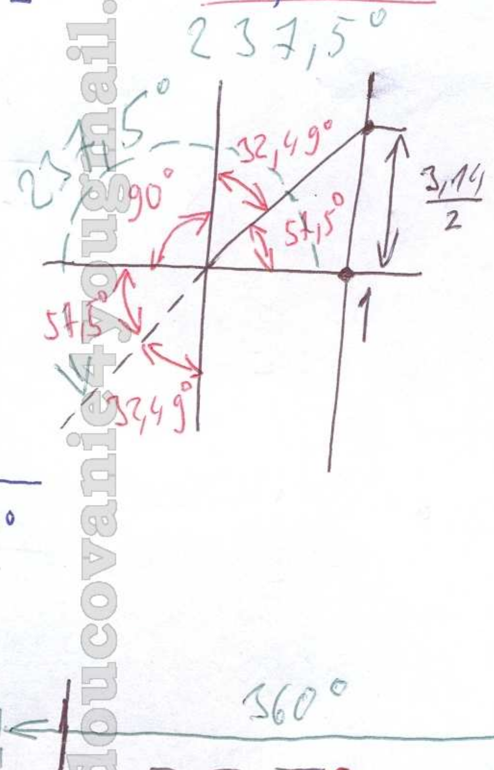
$$20 = 2\pi X_0 \cos\alpha \quad \mathrm{II.} \Rightarrow X_0 = \frac{20}{2\pi \cos\alpha}$$

$$\frac{5}{\sin\alpha} = \frac{20}{2\pi\cos\alpha}$$

$$10\pi\cos\alpha = 20\sin\alpha$$

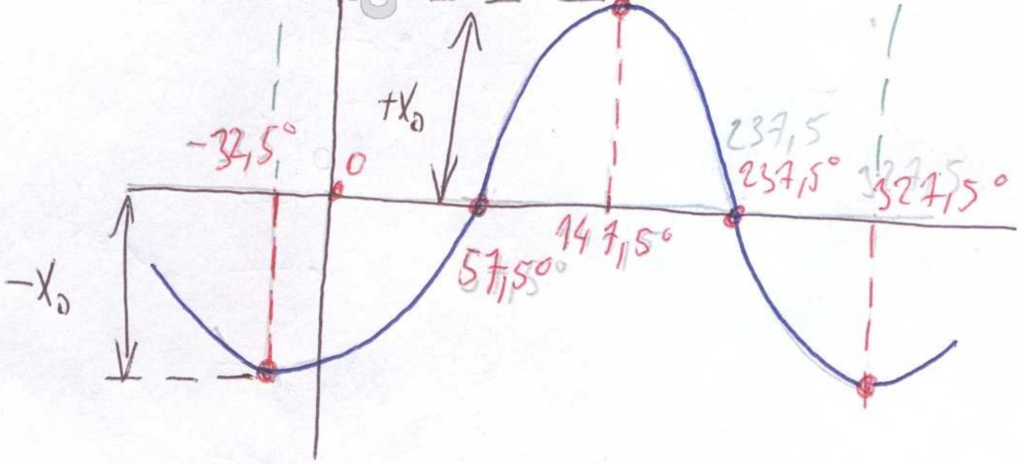
$$\frac{\pi}{2} = \frac{\sin\alpha}{\cos\alpha} = \operatorname{tg}\alpha$$

$$\alpha = \operatorname{arctg}\frac{\pi}{2} = \operatorname{arctg}\frac{3{,}14}{2} = 57{,}51^\circ$$



$$X_0 = \frac{5}{\sin\alpha} = \frac{5}{\sin 57{,}51^\circ}$$

$$X_0 = 5{,}9\ \text{cm}$$



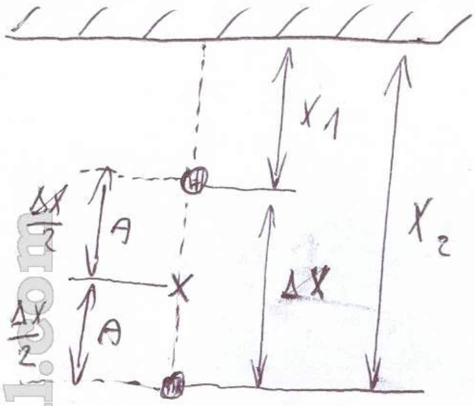
Častica harmonicky kmitá medzi polohami $X_1 = 5cm$ a $X_2 = 12$ cm , veľkosť maximálnej rýchlosti je 4,5 $m\ s^{-1}$. Vypočítajte frekvenciu a veľkosť maximálneho zrýchlenia.

$X_1 = 5\,cm$

$X_2 = 12\,cm$

$v_{max} = 4{,}5\,m\,s^{-1}$

$f,\ a_{max} = ?$



$X = A \sin \omega t$

$$v = \frac{dX}{dt} = \frac{d}{dt}(A \sin \omega t) = \omega A \cos(\omega t)$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(\omega A \cos(\omega t)) = -\omega^2 A \sin(\omega t)$$

$v_{max} = \omega A \qquad a_{max} = -\omega^2 A$

$\Delta X = X_2 - X_1 = 12 - 5 = 7$

$A = \frac{\Delta X}{2} = \frac{7}{2} = 3{,}5\,cm = 0{,}035\,m$

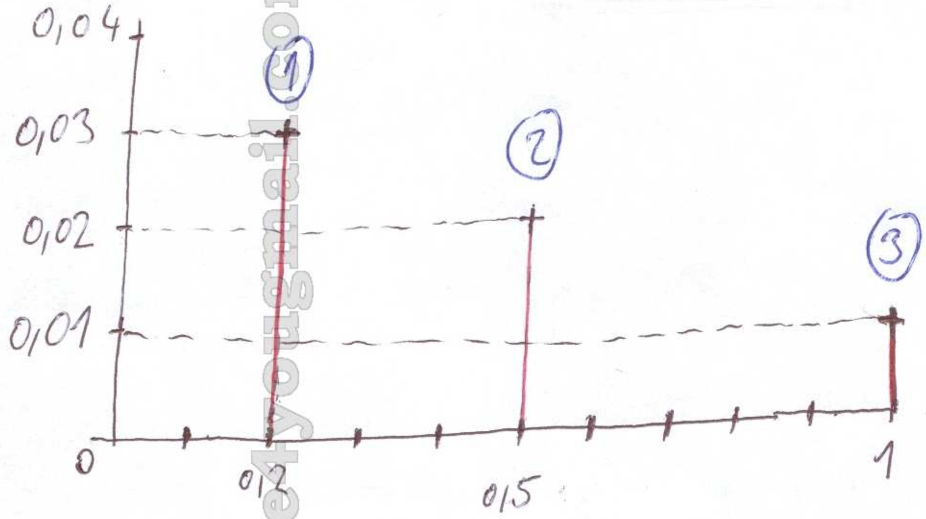
$\omega = 2\pi f$

$$v_{max} = \omega A \Rightarrow \omega = \frac{v_{max}}{A} = \frac{4{,}5}{0{,}035} = \underline{\underline{128{,}57\,s^{-1}}}$$

$$a_{max} = -\omega^2 A = -128,57^2 \cdot 0,035 = \underline{-578,56\ m\ s^{-2}}$$

Závislosť amplitúdy od frekvencie troch kmitavých pohybov ($X = A\sin\omega t$) je znázornená na obrázku. Napíšte rovnice ktoré popisujú závislosť výchylky týchto kmitavých pohybov od času a nakreslite ich časový priebeh.

$\underline{X = A.\sin\omega t}$



1

$f = 0,2\ s^{-1}$

$A = 0,03\ m$

$\omega = 2\pi f = 2\pi . 0,2 = 1,26\ s^{-1}$

$X = A\sin\omega t$

$\underline{X = 0,03\sin 1,26t}$

2

$f = 0,5\ s^{-1}$

$A = 0,02\ m$

$\omega = 3,14\ s^{-1}$

$X = A\sin\omega t$

$\underline{X = 0,02\sin 3,14t}$

③

$f = 1\,s^{-1}$

$A = 0{,}01\,m$

$\omega = 6{,}28\,s^{-1}$

$x = 0{,}01 \sin 6{,}28t$

Aká je frekvencia netlmeného harmonického pohybu častice hmotnosti 2g, ak amplitúda je 10 cm a celková energia 1 J ?

$m = 2g = 0{,}02\,kg$

$A = 10\,cm = 0{,}1\,m$

$E = 1\,J$

$f = ?$

$\omega = 2\pi f$

$x = A \sin \omega t$

$v = \omega A \cos \omega t$

$\omega A = v_{max}$

$E = \frac{1}{2} m v_{max}^2 = \frac{1}{2} m \omega^2 A^2$

v_{max}^2

ω^2

$E = \frac{1}{2} m 4\pi^2 f^2 A^2 \;/ \cdot 2 \;/ \cdot \frac{1}{4\pi^2 A^2 m}$

$\frac{2E}{4\pi^2 A^2 m} = f^2 \Rightarrow f = \sqrt{\frac{2E}{4\pi^2 A^2 m}} = \sqrt{\frac{2 \cdot 1}{4\pi^2 \cdot 0{,}1^2 \cdot 0{,}02}} = 50{,}32\,s^{-1}$

5. Špeciálna teória relativity

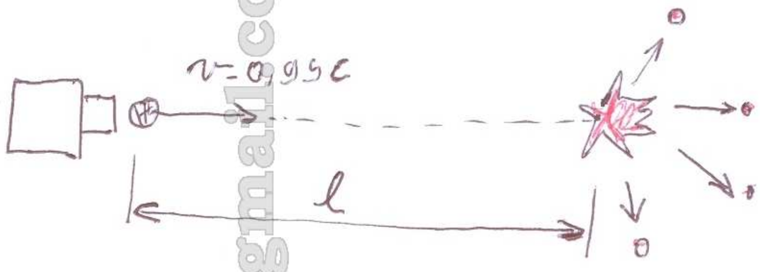
Častica pohybujúca sa rýchlosťou v = 0,99c vo vzťažnej sústave S, prešla od miesta svojho vzniku do miesta rozpadu vzdialenosť l=3km. Určite vlastnú dobu života tejto častice τ_0.

$v = 0{,}99c$

$l = 3\,km = \Delta s$

$\tau = ?$

Δt



$v = 0{,}99c = 0{,}99 . 300\,000\,km\,s^{-1} = 297\,000\,km\,s^{-1}$

$\Delta s = v . \Delta t$

$\Delta t = \frac{\Delta s}{v} = \frac{3\,km}{297\,000\,km\,s^{-1}} = 1{,}01 . 10^{-5}\,s$

$\tau = \Delta t \sqrt{1 - \frac{v^2}{c^2}} = 1{,}05 . 10^{-5} \sqrt{1 - \frac{0{,}99^2 c^2}{c^2}}$

$\tau = 1{,}48 . 10^{-6}\,s$

Raketa sa vzďaľuje od Zeme rýchlosťou 0,98c. Za akú dobu podľa pozemského pozorovateľa vykoná sekundová ručička palubných hodín jednu otáčku ?

$v = 0{,}98\,c$

$t = 60\,s = 1\,min$

$t_0 = ?$

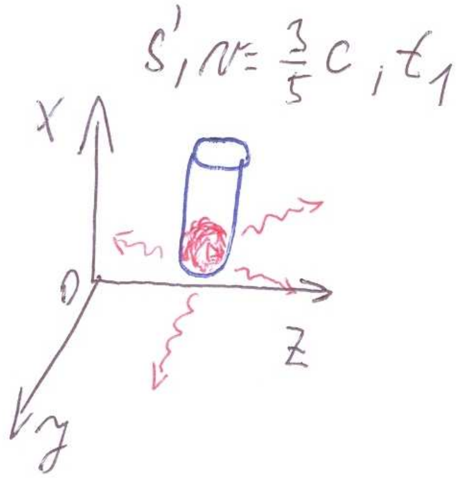
$$t_0 = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1\,min}{\sqrt{1 - \frac{(0{,}98c)^2}{c^2}}} = 5{,}025\,min$$

V súradnej sústave pozorovateľa S‘ je radioaktívny preparát. Ktorý sa podľa pozorovateľa v S‘ rozpadne za 12s. Pozorovateľ v S tvrdí , že sa súradnicová sústava pozorovateľa S‘ od neho vzďaľuje rýchlosťou v = (3/5)c. Akú dobu rozpadu preparátu nameria pozorovateľ v S ?

$t_1 = 12\,s$

$v = \frac{3}{5}c$

$t_2 = ?$



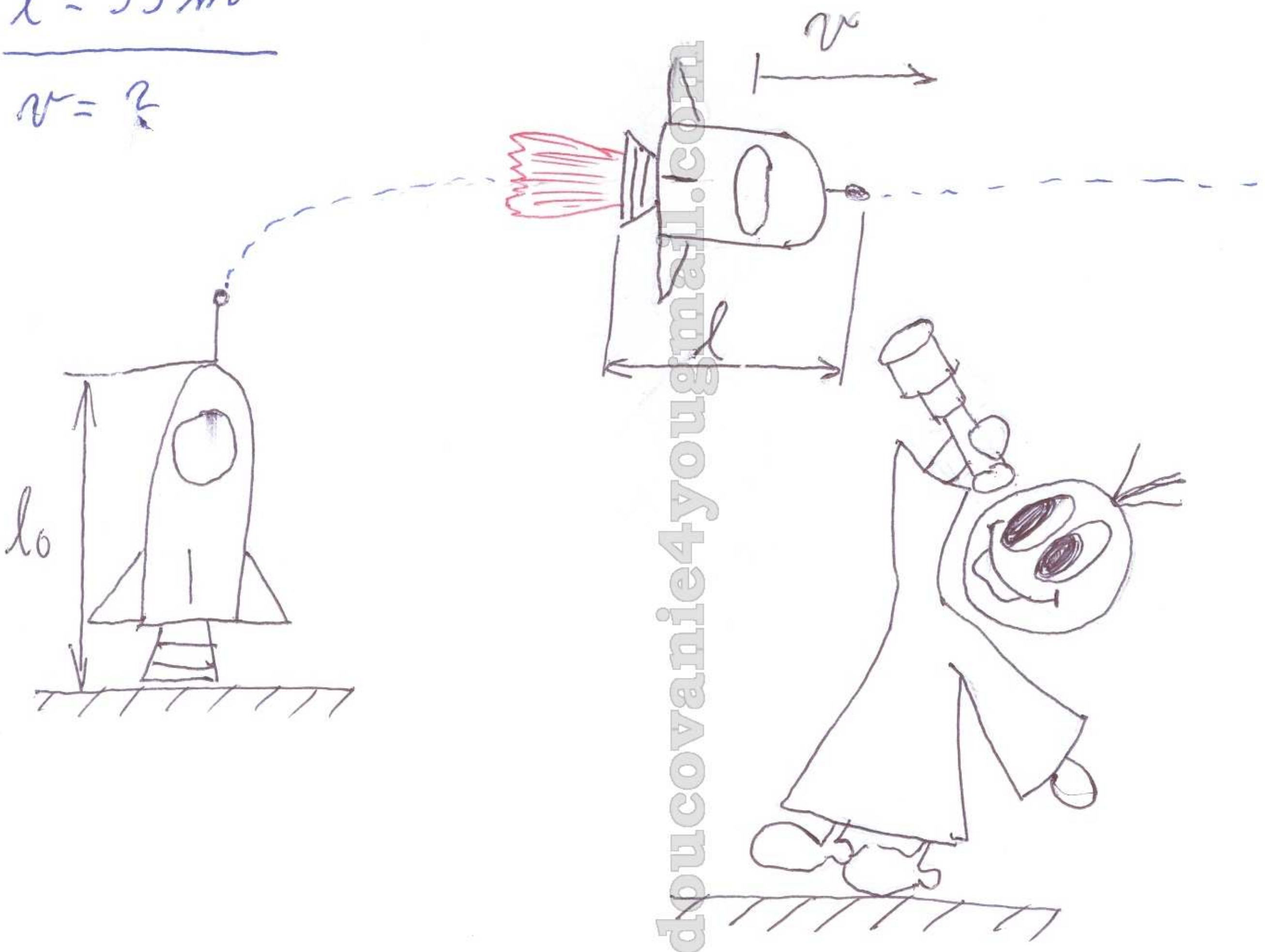
$$t_2 = \frac{t_1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{12}{\sqrt{1-\frac{\left(\frac{3}{5}c\right)^2}{c^2}}} = 15\,s$$

Raketa ja na Zemi dlhá 100m, ale počas letu pozemský pozorovateľ zistí, že jej dĺžka je 99m. Aká je rýchlosť rakety ?

$l_0 = 100\,m$

$l = 99\,m$

$v = ?$



$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{l}{l_0} = \sqrt{1 - \frac{v^2}{c^2}} = \frac{99}{100} = 0{,}99$$

$$0{,}99 = \sqrt{1-\frac{v^2}{c^2}} \Big/^{2}$$

$$\boxed{\sqrt{a} = (a)^{\frac{1}{2}}}$$

$$(0{,}99)^2 = 1-\frac{v^2}{c^2}$$

$$\sqrt{\left(-(0{,}99)^2+1\right).c^2} = v = \left[-(0{,}99^2)+1)\,c^2\right]^{\frac{1}{2}}$$

$v = \underline{\underline{42000 \text{ km s}^{-1}}}$

Akú rýchlosť musí mať pohybujúce sa teleso, aby sa jeho rozmery v smere pohybu zmenšili na polovicu ?

l_0

$l = \frac{1}{2} l_0$

$v = ?$

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{\frac{1}{2}\cancel{l_0}}{\cancel{l_0}} = \sqrt{1 - \frac{v^2}{c^2}} \quad /^2$$

$$0{,}25 = 1 - \frac{v^2}{c^2} \quad / \cdot (-1)$$

$$\sqrt{-0{,}25 + 1} = \frac{v}{c} = \sqrt{0{,}75} = 0{,}866$$

$$\underline{\underline{v = 0{,}866\,c}}$$

Určíte vlastnú dĺžku tyče pohybujúcej sa rýchlosťou c/2 vzhľadom na sústavu S, ak jej dĺžka v tejto sústave je 1m a uhol medzi vektorom rýchlosti a tyčou je 45^0.

$v = 0{,}5c$

$l = 1\,m$

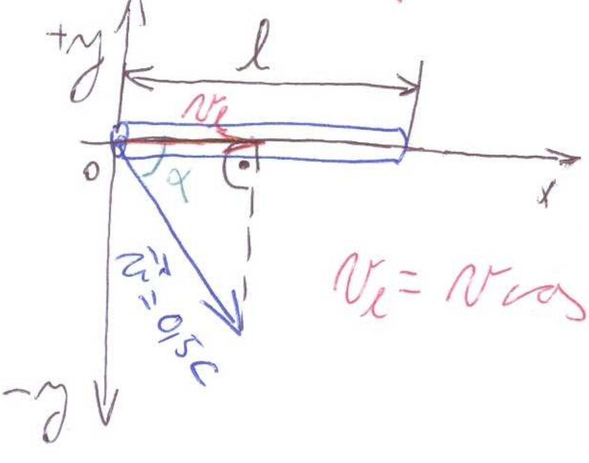
$\alpha = 45^0$

$$l = l_0\sqrt{1-\frac{v_l^2}{c^2}}$$

$$l_0 = \frac{l}{\sqrt{1-\frac{v_l^2}{c^2}}} = \frac{l}{\sqrt{1-\frac{(0{,}5c\cos\alpha)^2}{c^2}}} =$$

$(0{,}5c\cos\alpha) \rightarrow v_l$

$$= \frac{l}{\sqrt{1-\frac{0{,}5^2\cancel{c^2}\cos^2\alpha}{\cancel{c^2}}}} = \frac{1}{\sqrt{1-0{,}25\cos^2 45^0}} = \underline{\underline{1{,}07\,m}}$$



$v_l = v\cos\alpha$

Fyzik chce umiestniť detektor častíc v takej vzdialenosti od zdroja, aby sa väčšina častíc rozpadla práve na tomto mieste. Častice sa pohybujú rýchlosťou 0,99c a stredná doba života meraná v pokojovej sústave je 10^{-10} s .V akej vzdialenosti od zdroja je potrebné detektor umiestniť.

$v = 0{,}99c$

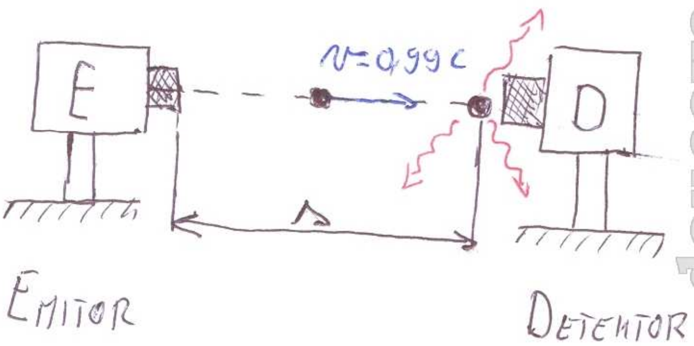
$t_0 = 1 \cdot 10^{-10}\,s$

$s = ?$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1 \cdot 10^{-10}}{\sqrt{1 - 0{,}99^2}} = 7{,}08 \cdot 10^{-10}\,s$$

$$s = v \cdot t = 0{,}99c \cdot t = 0{,}99 \cdot 300000000 \cdot 7{,}08 \cdot 10^{-10}$$

$$s = 0{,}21\,m = \underline{\underline{21\,cm}}$$



Pri akej rýchlosti telesa vzrastie jeho hustota o 10 %

$\Delta\rho = 10\%$

$v = ?$

$$l = l_0\sqrt{1-\frac{v^2}{c^2}} \quad \Big/ \cdot S \quad \Rightarrow \quad Sl = Sl_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad \boxed{V = V_0\sqrt{1-\frac{v^2}{c^2}}}$$

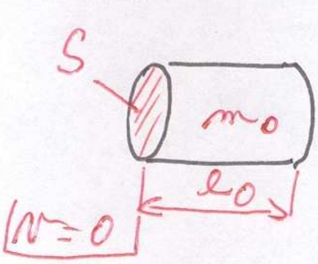
$$\boxed{\rho = 1{,}1 \cdot \rho_0}$$

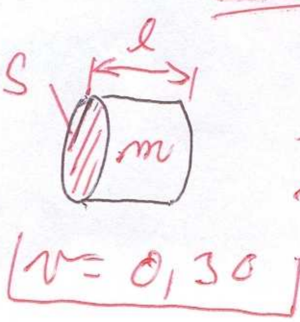
$$\rho = \frac{m}{V} \qquad \frac{\rho}{\rho_0} = \frac{\frac{m}{V}}{\frac{m_0}{V_0}} = \frac{m}{m_0}\,\frac{V_0}{V} = 1{,}1$$

$$\frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \frac{V_0}{V} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{m}{m_0}\,\frac{V_0}{V} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 1{,}1$$

$$\sqrt{1-\left(\frac{1}{1{,}1}-1\right)}\; c = v = \underline{0{,}3\,c}$$





$l_0 > l$ $\qquad S = \text{konšt}$

$m_0 < m$

Kinetická energia letiaceho elektrónu je 1MeV. Vypočítajte akou rýchlosťou sa pohybuje. (1eV=1,602.10^-19^J m_e=3,1.10^-31^kg)

$E_k = 1\ MeV$

$1eV = 1{,}602 \cdot 10^{-19}\ J$

$m_e = 9{,}1 \cdot 10^{-31}\ kg$

$v = ?$

$$E_k = \frac{\frac{1}{2} m v^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad v = xc$$

$$E_k = \frac{\frac{1}{2} m x^2 c^2}{\sqrt{1-\frac{x^2 \cancel{c^2}}{\cancel{c^2}}}}$$

$$2E_k\sqrt{1-x^2} = m x^2 c^2$$

$$\sqrt{1-x^2} = \frac{m x^2 c^2}{2E_k} \Big/ ^2$$

$$1-x^2 = \frac{m^2 x^4 c^4}{4E_k^2} \qquad x^2 = B$$

$$1-x^2 = \frac{m^2 B^2 c^4}{4E_k^2} \Big/ 4E_k^2$$

$$-m^2c^4B^2 - 4BE_k^2 + E_k^2 = 0 \Big/ \cdot(-1)$$

$$\underbrace{m^2c^4}_{a}B^2 + \underbrace{4BE_k^2}_{b} \underbrace{- E_k^2}_{c}$$

$$D = b^2 - 4ac = 4E_k^2(4E_k^2 + m^2)$$

$b = 4E_k^2$

$a = c^4 m^2$ $\qquad B_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

$c = -E_k^2$

$$E_{k(J)} = 1 \cdot 10^6 \cdot 1{,}602 \cdot 10^{-19} = 1{,}602 \cdot 10^{-13}\ J$$

$$B_{12} = \frac{-4E_k^2 \pm 2E_k\sqrt{4E_k^2 + m^2}}{2m^2c^4}$$

$$B_{12} = \begin{cases} 0{,}816 \\ \cancel{-16{,}176} \end{cases}$$

$B = x^2 \qquad x = \sqrt{B} = \sqrt{0{,}816}$

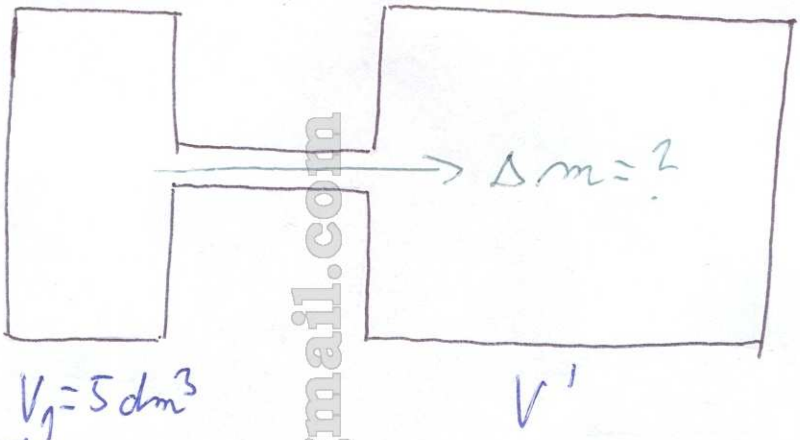
$x = 0{,}9$

$v = xc$

$v = 0{,}9c$

6. Náuka o teple

Vzduch v nádobe s objemom V = 5dm^3 sa nachádza pri teplote t =27^{0}C a tlaku p_1 = 2Mpa. Akú hmotnosť Δm bude mať vzduch, ktorý treba s nádoby vypustiť, aby tlak v nádobe klesol na hodnotu p_2 = 1MPa pri nezmenenej teplote.



$V_1 = 5\ dm^3$

$t = 27°C$

$p_1 = 2\ MPa$

$p_2 = 1\ MPa$

$\Delta t = 0$

$c_V = 728\ J\,kg^{-1}K^{-1}$

$\varkappa = 1{,}4$

$\Delta m = ?$

$T = t + 273{,}15 = 27 + 273{,}15 = 300{,}15\ K$

$\varkappa = \frac{c_p}{c_V} \qquad \varkappa c_V = c_p = 1{,}4 \cdot 728 = 1019{,}2$

$T = konšt \quad \Delta T = 0$

ADIABATICKÝ PROCES

$5\ dm^3 = 5 \cdot 10^{-3}\ m^3$

$\boxed{p_1 V_1^{\varkappa} = p_2 V_2^{\varkappa}}$

$V_2^{\varkappa} = \frac{p_1 V_1^{\varkappa}}{p_2} = \frac{2 \cdot 10^6 \cdot (5 \cdot 10^{-3})^{1,4}}{1 \cdot 10^6} = 1{,}2 \cdot 10^{-3} = V_2^{\varkappa}$

$$1{,}2 \cdot 10^{-3} = V_2^{\varkappa} \Big/ \log$$

$$\log(1{,}2 \cdot 10^{-3}) = \log V_2^{\varkappa}$$

$$\log(1{,}2 \cdot 10^{-3}) = \varkappa \log V_2$$

$$\log V_2 = \frac{\log(1{,}2 \cdot 10^{-3})}{\varkappa}$$

$$10^{\frac{\log(1{,}2 \cdot 10^{-3})}{\varkappa}} = V_2$$

$$V_2 = 10^{-2{,}08} = 8{,}31 \cdot 10^{-3}\,m^3$$

PRE VZDUCH PLATÍ:

$p_0 = 0{,}1\,MPa$

$t_0 = 0\,°C \quad T_0 = 273{,}15\,K$

$\rho_0 = 1{,}293\,kg\,m^{-3}$

$$pV = \frac{m}{M_m} R_m T \Rightarrow m = \frac{pVM_m}{R_m T}$$

$$\rho = \frac{m}{V} = \frac{1}{\cancel{V}} \cdot \frac{p\cancel{V}M_m}{R_m T} = \frac{pM_m}{R_m T}$$

$$\rho_1 = \frac{p_1 M_m}{R_m T_1}$$

$$\rho_0 = \frac{p_0 M_m}{R_m T_0}$$

$$\frac{\rho_1}{\rho_0} = \frac{\frac{p_1 M_m}{R_m T_1}}{\frac{p_0 M_m}{R_m T_0}}$$

$$\rho_1 = \rho_0 \frac{p_1 T_0}{T_1 p_0} = 1{,}293 \cdot \frac{2 \cdot 10^6 \cdot 273{,}15}{300{,}15 \cdot 0{,}1 \cdot 10^6}$$

$$\rho_1 = 23{,}53\ kg\, m^{-3}$$

$$m_1 = \rho_1 V_1 = 23{,}53 \cdot 5 \cdot 10^{-3} = 0{,}127\ kg$$

$$V_1 : V' = m^* : \Delta m \qquad m_1 = m^* + \Delta m$$

$$\frac{V'}{V_1 + V'} \cdot m_1 = \Delta m = 0{,}05\ kg$$

$$V_2 - V_1 = V' = 3{,}31 \cdot 10^{-3}\ m^3$$

Koľko molekúl je v guľovej nádobe polomeru 3cm, naplnenej kyslíkom, keď jeho teplota je $t = 27^0C$ a tlak 133.10^{-4} Pa ? ($M = 32.10^{-3}$ kg mol^{-1} , $R = 8{,}314$ JK^{-1} mol^{-1} , $N_A = 6{,}022.10^{23}$ mol^{-1})

$R = 3\,cm = 3.10^{-2}\,m$

$t = 27\,°C$

$p = 133.10^{-4}\,Pa$

$M = 32.10^{-3}\,kg\,mol^{-1}$

$R = 8{,}314\,JK^{-1}$

$N_A = 6{,}022.10^{23}\,mol^{-1}$

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (3.10^{-2})^3 = 1{,}13.10^{-4}\,m^3$$

$$T = t + 273{,}15 = 27 + 273{,}15 = 300{,}15\,K$$

$$pV = \frac{m}{M}RT$$

$$m = \frac{pVM}{RT} = \frac{1{,}33.10^{-4}.1{,}13.10^{-4}.32.10^{-3}}{8{,}314.300{,}15} = 1{,}92.10^{-13}\,kg$$

$$N = \frac{m}{M} = \frac{1,92 \cdot 10^{-13}}{32 \cdot 10^{-3}} = 6 \cdot 10^{-12}$$

$$n = N \cdot N_A = 6 \cdot 10^{-12} \cdot 6,022 \cdot 10^{23} = \underline{3,61 \cdot 10^{12}}$$

Za predpokladu, že priemerná teplota Slnka je 2.10^7 K, vypočítajte aká je stredná kinetická energia postupného pohybu atómov vo vnútri Slnka ! ($k = 1,38.10^{-23} JK^{-1}$)

$k = 1,38 \cdot 10^{-23}\ JK^{-1}$

$T = 2 \cdot 10^7\ K$

$\bar{E}_k = ?$

$$\varepsilon = \frac{3}{2} kT = \frac{3}{2} 1,38 \cdot 10^{-23} \cdot 2 \cdot 10^7 = \underline{4,14 \cdot 10^{-16}\ J}$$

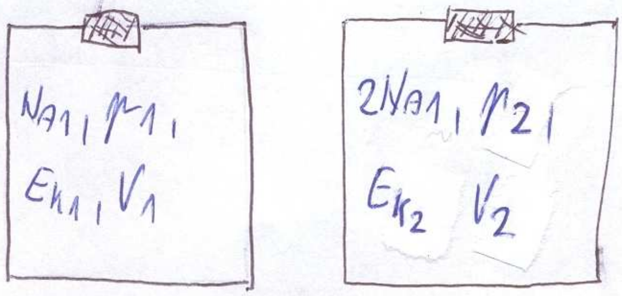
Ideálny plyn uzavretý v nádobe obsahuje N_A molekúl. Zdvojnásobíme teraz počet molekúl v tej istej nádobe tak, aby sa celková kinetická energia molekúl plynu pritom nezmenila. (Celková kinetická energia molekúl je v obidvoch prípadoch rovnaká)
a, Aký je podiel konečného a začiatočného tlaku plynu ?
b, Aký je podiel konečnej a začiatočnej teploty plynu ?

N_{A1}

$N_{A2} = 2N_{A1}$

$E_{k1} = E_{k2}$

$\frac{p_2}{p_1} = ?\quad \frac{T_2}{T_1} = ?$



$V_1 = V_2 \quad E_{k1} = E_{k2}$

$$p_1 = \frac{2}{3}\boxed{n_1 \bar{E}_{k_1}} \longrightarrow E_{k_1}$$

$$E_{k_1} = E_{k_2}$$

$$p_2 = \frac{2}{3}\boxed{2n_1 \bar{E}_{k_2}} \longrightarrow E_{k_2}$$

$$\frac{\cancel{3}p_1}{\cancel{3}p_2} = \frac{\cancel{2}\cancel{\boxed{n_1 \bar{E}_{k_1}}}}{\cancel{2}\cdot 2\cancel{\boxed{n_1 \bar{E}_{k_2}}}}$$

$$\boxed{\frac{p_1}{p_2} = 1}$$

$$p_1 V_1 = N_{A_1} k T_1$$

$$p_2 V_2 = 2N_{A_1} k T_2$$

$$\frac{p_1 \cancel{V_1}}{p_2 \cancel{V_2}} = \frac{\cancel{N_{A_1}}\cancel{k}T_1}{2\cancel{N_{A_1}}\cancel{k}T_2} \qquad V_1 = V_2$$

$$\frac{p_1}{p_2} = \frac{T_1}{2T_2} = 1$$

$$\frac{T_1}{2T_2} = 1 \qquad \boxed{\frac{T_2}{T_1} = \frac{1}{2}}$$

Vypočítajte aká je vnútorná energia m= 10g dusíka teploty 30^0C ! Aké časť tejto energie pripadá postupný a aká na rotačný pohyb molekúl ? ($M = 28.10^{-3}$ kg mol^{-1} , $R = 8,314$ JK^{-1} mol^{-1})

N_2

$M = 28 \cdot 10^{-3}$ kg mol

$m = 10g = 0{,}01$ kg

$t = 30°C$

$N_A = 6{,}022 \cdot 10^{23}$

$U_P, U_R, U = ?$

$i = 3 + 2 = 5$

$\bar{E} = \frac{i}{2} kT$

$\bar{E}_P = \frac{3}{2} kT$

$\bar{E}_R = \frac{2}{2} kT$

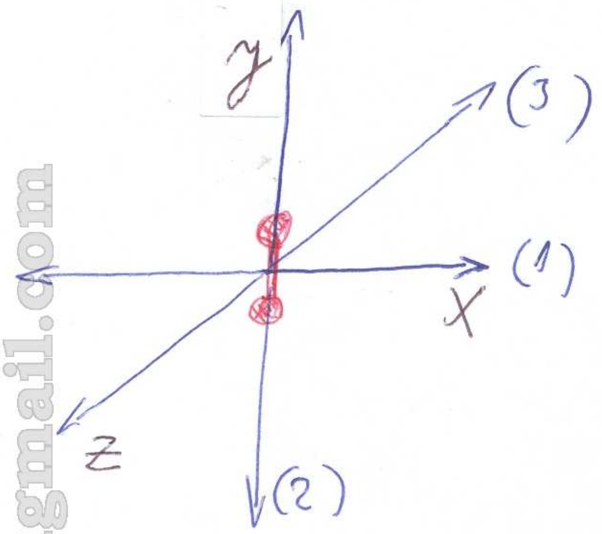
$\bar{E} = \frac{3+2}{2} kT$

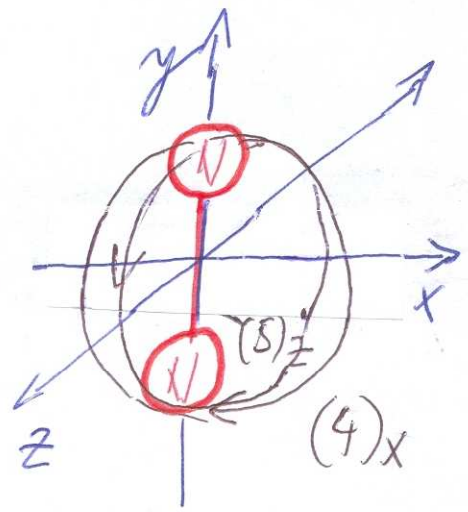
$U_P = n \bar{E}_P$

$U_R = n \bar{E}_R$

$U = n \bar{E}$

$N = \frac{m}{M}$





$T = t + 273{,}15 = 303{,}15$ K

$k = 1{,}38 \cdot 10^{-23}$ JK^{-1}

$$n = N . N_A = \frac{m}{M} N_A$$

$$U = \frac{5}{2} \underbrace{\boxed{\frac{m}{M} N_A}}_{n} kT = \frac{5}{2} \frac{10.10^{-3}}{28.10^{-3}} \cdot 6{,}022.10^{23} \cdot 1{,}38.10^{-23} \cdot 303{,}15$$

$$\underline{U = 2249{,}36\ J}$$

$$U_R = \frac{2}{2} \frac{m}{M} N_A kT = \underline{899{,}744\ J}$$

$$U_P = \frac{3}{2} \frac{m}{M} N_A kT = \underline{1349{,}6\ J}$$

Vypočítajte ako sa zmení stredná hodnota kinetickej energie, molekúl argónu, ktorého hmotnosť je m = 200g. Keď pri zachovaní stáleho objemu dodáme mu dodáme teplo Q = 3516 J .

$m = 200\,g = 0{,}2\,kg$

$\Delta V = 0$

$\Delta Q = 3516\,J$

$M = 40.10^{-3}\,kg\,mol^{-1}$

$N_A = 6{,}022.10^{23}\,mol^{-1}$

$\Delta \bar{E} = ?$

$$n = \frac{m}{M} = \frac{0{,}2}{40.10^{-3}} = 5\,mol$$

$N = n N_A = 5 \cdot 6{,}022 \cdot 10^{23}\ mol^{-1} = 3 \cdot 10^{24}$

$\Delta V = 0 \Rightarrow W = 0$ PRÁCA PLYNU

$\Delta \bar{E} = \dfrac{\Delta Q}{N} = \dfrac{3516}{3 \cdot 10^{24}} = 1{,}16 \cdot 10^{-21}\ J$

Pri akej teplote je stredná kvadratická rýchlosť molekúl dusíka práve polovičná ako pri izbovej teplote $t = 20^0C$?

$t_0 = 20°C$

$t = ?$

$T_0 = 273{,}15 + 20 = 293{,}15\ K$

$t_0 \rightarrow \bar{E}_0$

$t \rightarrow E = \dfrac{\bar{E}_0}{2}$

$\bar{E}_0 = \dfrac{i}{2} k T_0$

$\dfrac{\bar{E}_0}{2} = \bar{E} = \dfrac{i}{2} k T$

$\dfrac{\bar{E}_0}{\frac{\bar{E}_0}{2}} = \dfrac{\frac{i}{2} k T_0}{\frac{i}{2} k T}$

$$\frac{2\cancel{\bar{E}_0}}{\cancel{\bar{E}_0}} = \frac{T_0}{T}$$

$$\frac{T_0}{T} = 2 \qquad T = \frac{T_0}{2} = \frac{293,15}{2} = 146,57\,K$$

$$t = 146,57 - 273,15 = -126\,^\circ C$$

Stredná kvadratická rýchlosť molekúl plynu je $v_s = 800ms^{-1}$. Koľko molekúl obsahuje 1 kg toho plynu, ak jeho teplota je 27^0C ?

$\bar{v}_s = 800\,m\,s^{-1}$

$m = 1\,kg$

$t = 27\,^\circ C$

$N = ?$

$k = 1,38 . 10^{-23}\,JK^{-1}$

$T = 273,15 + 27 = 300,15\,^\circ K$

$$\bar{E} = \frac{3}{2} kT = \frac{3}{2} 1,38 . 10^{-23} . 300,15 = 6,21 . 10^{-21}\,J$$

$$E_c = \frac{1}{2} m v^2 = \frac{1}{2} 1 . 800^2 = 320 . 10^3\,J$$

$$n = \frac{E_c}{\bar{E}} = \frac{320 . 10^3}{6,21 . 10^{-21}} = 5,15 . 10^{25}$$

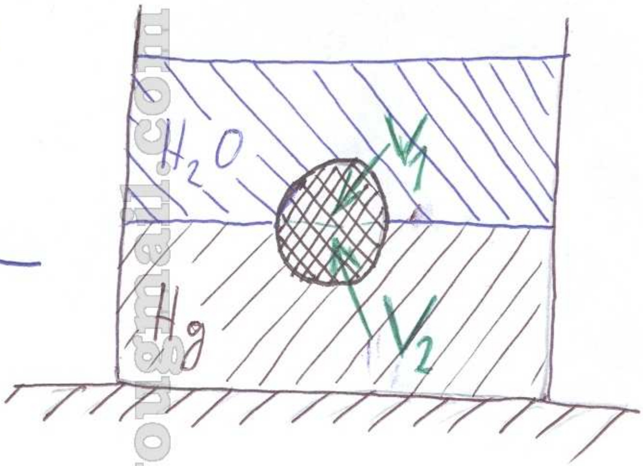
7. Mechanika tekutín

Do nádoby, v ktorej je naliata ortuť a voda, hodíme oceľovú guľku. Aká časť objemu guľky bude vo vode ? Hustota ocele je **7,8.10³ kg.m⁻³**, ortuti **13,6.10³ kg.m⁻³** a vody **10³ kg.m⁻³**

$\rho_l = 7{,}8 \cdot 10^3\ kg\ m^{-3}$

$\rho_{Hg} = 13{,}6 \cdot 10^3\ kg\ m^{-3}$

$\rho_V = 1000\ kg\ m^{-3}$



$F = V \rho g$

$\vec{F_V} = \vec{F_{1V}} + \vec{F_{2V}}$

$F_V = V_1 \rho_V g + V_2 \rho_{Hg} g$

$F_g = (V_1 + V_2) \rho_l \cdot g$

$|\vec{F_g}| = |\vec{F_V}|$

$(V_1 + V_2) \rho_l \cdot g = V_1 \rho_V g + V_2 \rho_{Hg} g$

$V_1 (\rho_l - \rho_V) = V_2 (\rho_{Hg} - \rho_l)$

$$\frac{V_1}{V_2} = \frac{\rho_{Hg} - \rho_l}{\rho_l - \rho_v} = \frac{13{,}6 \cdot 10^3 - 7{,}8 \cdot 10^3}{7{,}8 \cdot 10^3 - 1000} = 0{,}85$$

$$\frac{V_1}{V_2} = \frac{5{,}8}{6{,}8} \qquad 6{,}8\,V_1 = 5{,}8\,V_2$$

$$\frac{V_1}{V} = \frac{5{,}8}{5{,}8 + 6{,}8} = \underline{0{,}46}$$

Vodorovnou trubicou nerovnakého prierezu preteká voda. Treba určiť aké množstvo vody **Q** preteká každým prierezom trubice za **1 sekundu** keď v miestach s prierezom **S_1 = 10cm²** resp. s prierezom **S_2 = 20cm²** umiestnené manometrické trubice ukazujú rozdiel vodných hladín **Δh = 20 cm** .

$\rho = 1000\ kg\,m^{-3}$

$S_1 = 10\ cm^2$

$S_2 = 20\ cm^2$

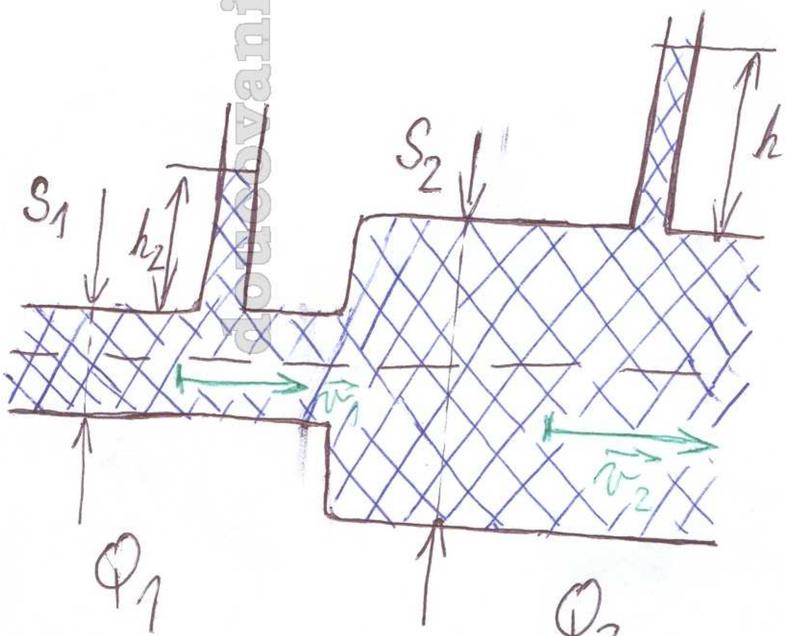
$\Delta h = 20\ cm$

$Q = ?$

$h_1 - h_2 = \Delta h$

$Q_1 = S_1 v_1$

$Q_2 = S_2 v_2$



$$Q_1 = Q_2$$

$$S_1 v_1 = S_2 v_2$$

$$\frac{1}{2}\rho v_1^2 + p_1 = \frac{1}{2}\rho v_2^2 + p_2$$

$$\frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = p_2 - p_1$$

$$\frac{1}{2}\cancel{\rho}(v_1^2 - v_2^2) = \cancel{\rho} g(h_2 - h_1) \Big/ \cdot 2$$

I. $v_1^2 - v_2^2 = 2g\Delta h$

II. $v_1 S_1 = v_2 S_2$ $\quad v_1 = \dfrac{v_2 S_2}{S_1}$

$$\left(\frac{v_2 S_2}{S_1}\right)^2 - v_2^2 = 2g\Delta h$$

$$\frac{v_2^2 S_2^2}{S_1^2} - v_2^2 = 2g\Delta h$$

$$\frac{v_2^2 S_2^2}{S_1^2} - v_2^2 = 2g\Delta h$$

$$v_2 = \sqrt{\frac{2g\Delta h}{\left(\frac{S_2^2}{S_1^2} - 1\right)}} = \sqrt{\frac{2 \cdot 9{,}8 \cdot 0{,}2}{\frac{(20 \cdot 10^{-2})^2}{(10 \cdot 10^{-2})^2} - 1}} = 1{,}14\, m\, s^{-1}$$

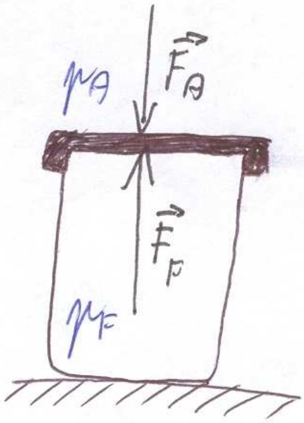
$$Q = S_2 v_2 = 0{,}2 \cdot 1{,}14 = \underline{\underline{0{,}228\, m^3 s^{-1}}}$$

Vo vnútri zaváracej fľaše pôsobí tlak vodnej pary **1962 Pa**, z vonka pôsob tlak vzduchu **$0{,}98.10^5$ Pa**. Vnútorný priemer hrdla zaváracej fľaše je **90mm**. Aká tlaková sila uzatvára kryt fľaše ?

$p_F = 1962\, Pa$

$p_A = 0{,}98 \cdot 10^5\, Pa$

$d = 90\, mm \quad r = 45\, mm = 45 \cdot 10^{-3}\, m$



$F = ?$

$S = \pi r^2 = \pi (45 \cdot 10^{-3})^2 = 6{,}36 \cdot 10^{-3}\, m^2$

$F_F = p_F \cdot S \qquad F_A = p_A \cdot S$

$\vec{F} = \vec{F}_A - \vec{F}_F = S(p_A - p_F) = 6{,}36 \cdot 10^{-3} \cdot (0{,}98 \cdot 10^5 - 1962) = 610{,}8\, N$

V určitej výške nad zemským povrchom namerali tlak vzduchu **p = 50.65.10^3 Nm^{-2}** .
Nájdite výšku **h** keď pri zemskom povrchu je tlak **p_o = 101,3.10^3 Nm^{-2}** Hustota vzduchu za tohto tlaku je **ρ_o = 1,29 kgm^{-3}** a keď predpokladáme , že teplota vzduchu je všade rovnaká.

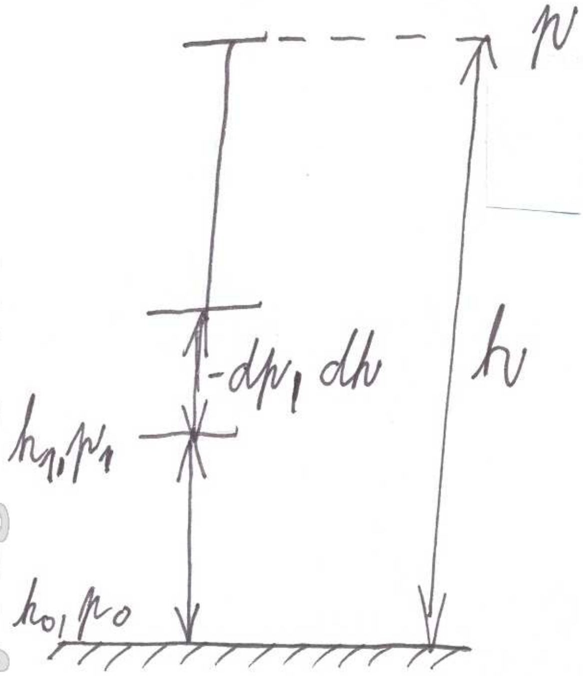
$h = ?$

$p_h = 50{,}63 \cdot 10^3\ Pa$

$p_0 = 101{,}3 \cdot 10^3\ Pa$

$\rho_0 = 1{,}29\ kg\ m^{-3}$

$t = konst$



$$-dp = \rho g\, dh$$

$$pV = R\frac{m}{M}T \quad / \frac{1}{V}$$

$$p = R\frac{m}{MV}T \qquad \rho = \frac{m}{V}$$

$$p = RT\rho \cdot \frac{1}{M} \Rightarrow \rho = p\frac{M}{RT}$$

$$-dp = \rho g\, dh$$

$$-dp = p\frac{M}{RT} g\, dh$$

$$-dp\,\frac{1}{p} = \frac{M}{RT}\,g\,dh \;\Big/ \int$$

$$-\int_{p_0}^{p} \frac{1}{p}\,dp = \int_{h_0}^{h} \frac{M}{RT}\,g\,dh$$

$$-\left[\ln p\right]_{p_0}^{p} = \frac{M}{RT}\,g\left[h\right]_{h_0}^{h}$$

$$-(\ln p - \ln p_0) = \frac{M}{RT}\,g(h-h_0) \;\Big/ \cdot(-1)$$

$$\ln\frac{p}{p_0} = \frac{-M}{RT}\,g(h-h_0)$$

$$\frac{p}{p_0} = e^{-\frac{M}{RT}g(h-h_0)} \;\Big/ \cdot p_0$$

$$p = p_0 e^{-\frac{M}{RT}g(h-\overset{0}{\overset{\shortparallel}{h_0}})} = p_0 e^{-\frac{M}{RT}gh}$$

$$\ln\frac{p}{p_0} = -\frac{M}{RT}gh + \frac{M}{RT}gh_0$$

$$\boxed{h_0 = 0}$$

$$\ln\frac{p}{p_0} - \frac{M}{RT}gh_0 = -\frac{M}{RT}gh \Big/ \cdot \frac{1}{-\frac{M}{RT}g}$$

$$h = \frac{\ln\frac{p}{p_0} - \frac{M}{RT}gh_0}{\frac{-Mg}{RT}}$$

$$h = \ln\frac{p}{p_0}\cdot\left(-\frac{RT}{Mg}\right) - \left(\frac{\cancel{Mg}}{\cancel{RT}}\cdot\left(\frac{\cancel{-RT}}{\cancel{M\cdot g}}\right)h_0\right)$$

$$h = -\ln\frac{p}{p_0}\cdot\frac{RT}{M\cdot g} + \cancel{h_0}$$

$$h = \ln\frac{p_0}{p}\,\frac{RT}{Mg} \Big/ \frac{1}{p_0} \qquad \rho_0 = p_0\frac{M}{RT}$$

$$\frac{h}{p_0} = \ln \frac{p_0}{p} \frac{RT}{Mg p_0} \rightarrow \frac{1}{\rho_0}$$

$$\frac{h}{p_0} = \ln \frac{p_0}{p} \frac{1}{\rho_0} \cdot \frac{1}{g} \Big/ \cdot p_0$$

$$h = \ln \frac{p_0}{p} \cdot \frac{p_0}{\rho_0 g}$$

$$h = \ln \frac{101{,}3 \cdot 10^3}{50{,}63 \cdot 10^3} \cdot \frac{101{,}3 \cdot 10^3}{1{,}29 \cdot 9{,}8} = \underline{\underline{5528{,}95\,m}}$$

Kúsok skla má tiaž **1,37 N**. Vo vode je jeho zdanlivá tiaž **0,824 N**. Aká je hustota skla ?

$G = 1{,}37\,N$

$G_2 = 0{,}824\,N$

$\rho_v = 1000\,kg\,m^{-3}$

$\rho_s = ?$

$G = V \rho_s g$

$V = \dfrac{G}{\rho_s g}$

$F_v = G - G_2 = V \rho_v g = \dfrac{G}{\rho_s g} \cdot \rho_v \cdot g = \dfrac{G \rho_v}{\rho_s}$

$G - G_2 = \dfrac{G \rho_v}{\rho_s}$

$\rho_s = \dfrac{G \rho_v}{G - G_2} = \dfrac{1{,}37 \cdot 1000}{1{,}37 - 0{,}824} = 2490\,kg\,m^{-3} = 2{,}49\,g\,cm^{-3}$

Dutá mosadzná guľa hmotnosti **0,3kg** sa ponorí do vody polovicou svojho objemu. Aký je jej vonkajší priemer a hrúbka steny, keď hustota mosadze je **8,4 g.cm^{-3}** ?

$m = 0,3\,kg$

$\rho = 8,4\,g\,cm^{-3}$

$d, h = ?$

$F_V = F_g = g\,m = 9,8 \cdot 0,3 = 2,94\,N$

$F_V = V_{1/2}\,\rho_V\,g$

$$V_{1/2} = \frac{F_V}{\rho_V \cdot g} = \frac{2,94}{1000 \cdot 9,8} = 3 \cdot 10^{-4}\,m^3$$

$V = V_{1/2} \cdot 2 = 6 \cdot 10^{-3}\,m^3$

$$V = \frac{4}{3}\pi R^3$$

$$\sqrt{\frac{3V}{4\pi}} = R = \sqrt[3]{\frac{3 \cdot 6 \cdot 10^{-4}}{4 \cdot \pi}} = 0,052\,m$$

$$m = \frac{4}{3}\pi\left(R^3 - R_0^3\right)\rho_M$$

$$3m = 4\pi R^3 \rho_M - 4\pi R_0^3 \rho_M$$

$$3m - 4\pi R^3 \rho_M = -4\pi R_0^3 \rho_M$$

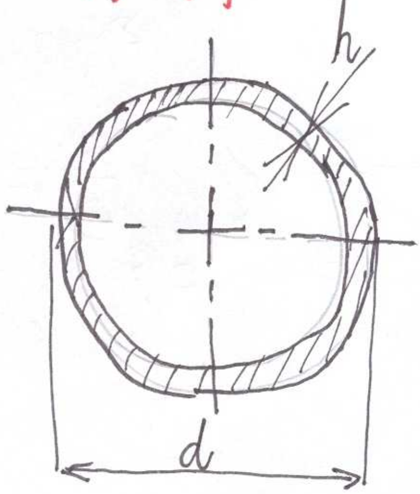
$$\sqrt[3]{\frac{3m - 4\pi R^3 \rho_M}{-4\pi \rho_M}} = R_0 =$$

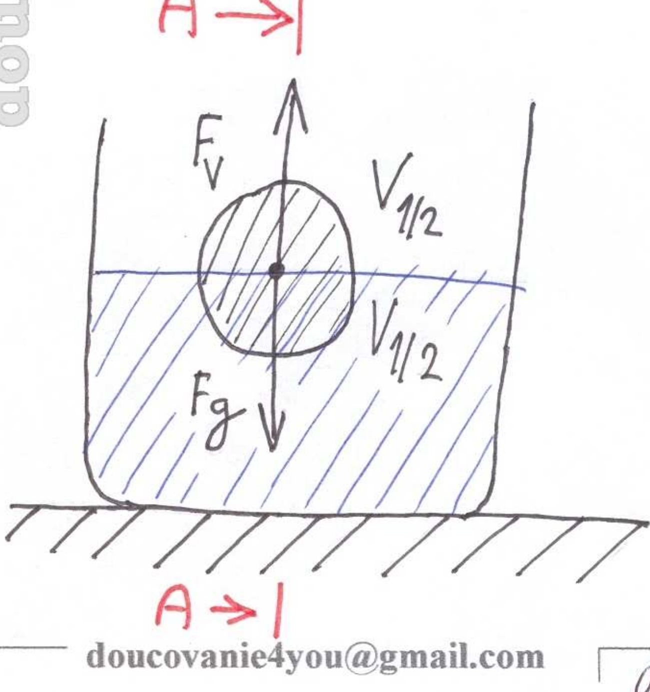
$$= \sqrt[3]{\frac{3 \cdot 0{,}3 - 4\pi (0{,}052)^3 \cdot 8400}{-4\pi \cdot 8400}} = 0{,}05\,m$$

$$h = R - R_0 = 0{,}51 - 0{,}05 = \underline{\underline{1 \cdot 10^{-3}\,m}}$$

$$d = 2R = 2 \cdot 0{,}052 = 0{,}104\,m = \underline{\underline{10{,}4\,cm}}$$

A - A



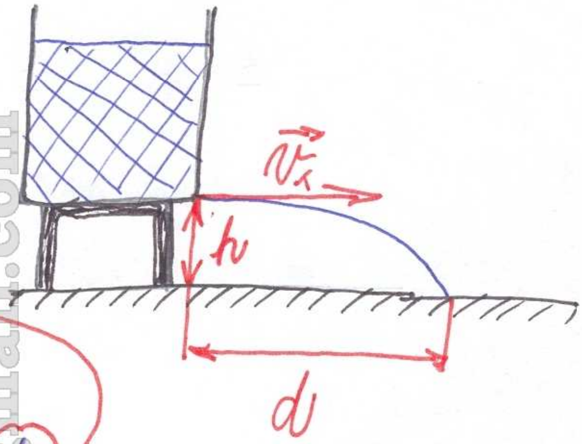


Akou rýchlosťou vyteká voda s rezervoára, keď z otvoru vo výške **h = 15 cm** nad vodorovnou rovinou strieka do vzdialenosti **d = 20 cm** na túto vodorovnú rovinu.

$h = 15\,cm = 0{,}15\,m$

$d = 20\,cm = 0{,}2\,m$

$v = ?$



$d = v_x t$

$h = \frac{1}{2} g t^2 \Rightarrow \sqrt{\frac{2h}{g}} = t$

$$d = v_x \sqrt{\frac{2h}{g}}$$

$$v_x = \frac{d}{\sqrt{\frac{2h}{g}}} = \frac{0{,}2}{\sqrt{\frac{2 \cdot 0{,}15}{9{,}8}}} = 1{,}143\,m\,s^{-1}$$

Do vody sú ponorené dve kapiláry s polomermi **r_1=1mm r_2=1,5mm**. Vypočítajte povrchové napätie vody, keď rozdiel hladín vodných stĺpcov v týchto kapilárach je **Δh = 4,9 mm** a keď predpokladáme , že voda dokonale zmáča stenu sklenenej trubice kapiláry.

$r_1 = 1 \cdot 10^{-3}\,m = 1\,mm$

$r_2 = 1{,}5 \cdot 10^{-3}\,m = 1{,}5\,mm$

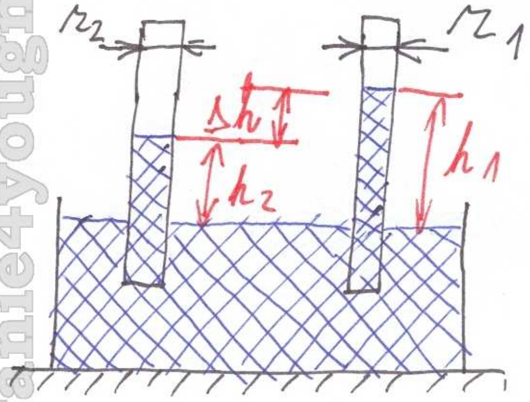
$\Delta h = 4{,}9 \cdot 10^{-3}\,m = 4{,}9\,mm$

$\sigma = ?$

$\sigma = \frac{1}{2} h \rho g r$

$\sigma = \frac{1}{2} h_1 \rho g r_1$

$\sigma = \frac{1}{2} h_2 \rho g r_2$



$r_1 < r_2 \Rightarrow h_1 > h_2$

$\frac{2\sigma}{g\rho r} = h \qquad h_1 = \frac{2\sigma}{g\rho r_1} \qquad h_2 = \frac{2\sigma}{g\rho r_2}$

$$h_1 - h_2 = \Delta h$$

$$\frac{2\sigma}{g\rho r_1} - \frac{2\sigma}{g\rho r_2} = \Delta h$$

$$\frac{2\sigma}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \Delta h$$

$$\sigma = \frac{g\rho\Delta h}{2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} = \frac{1000 \cdot 9{,}8 \cdot 4{,}9 \cdot 10^{-3}}{2\left(\frac{1}{1 \cdot 10^{-3}} - \frac{1}{1{,}5 \cdot 10^{-3}}\right)}$$

$$\sigma = 0{,}072\,Nm = \underline{\underline{72 \cdot 10^{-3}\,Nm}}$$

8. Gravitačné pole

V kovovej guli polomeru **R**, je vytvorená dutina **r=R/2**
Akou silou bude pôsobiť tento útvar na guľôčku hmotnosti **m**, ktorá je vo vzdialenosti **d** od stredu pôvodnej gule ak hmotnosť pôvodnej gule je **M**.

R

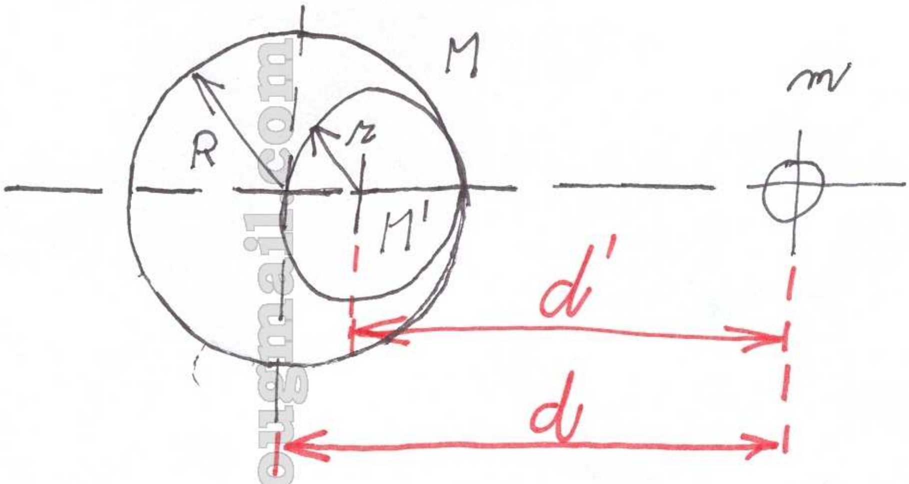
$r = \frac{R}{2}$

m

d

m

$F = ?$



$$F = F_1 - F_2 = \varkappa \frac{Mm}{d^2} - \varkappa \frac{M'm}{(d')^2}$$

$$d' = d - r$$

$$\frac{M'}{M} = \frac{V'}{V} = \frac{\cancel{\frac{4}{3}}\cancel{\pi} r^3}{\cancel{\frac{4}{3}}\cancel{\pi} R^3} = \frac{r^3}{R^3} \Rightarrow M' = M\frac{r^3}{R^3}$$

$$\varkappa \frac{Mm}{d^2} - \varkappa \frac{M\frac{r^3}{R^3}m}{(d-r)^2} = F =$$

$$= \mathcal{K}\frac{Mm}{d^2} - \frac{\mathcal{K}Mr^3m}{R^3(d-r)^2} =$$

$$= \mathcal{K}Mm\left(\frac{1}{d^2} - \frac{r^3}{R^3(R-r)^2}\right) =$$

$$\boxed{r = \frac{R}{2}}$$

$$= \mathcal{K}Mm\left(\frac{1}{d^2} - \frac{\left(\frac{R}{2}\right)^3}{R^3\left(d-\frac{R}{2}\right)^2}\right) =$$

$$= \mathcal{K}Mm\left(\frac{1}{d^2} - \frac{\frac{R^3}{8}}{R^3(d-0{,}5R)}\right)$$

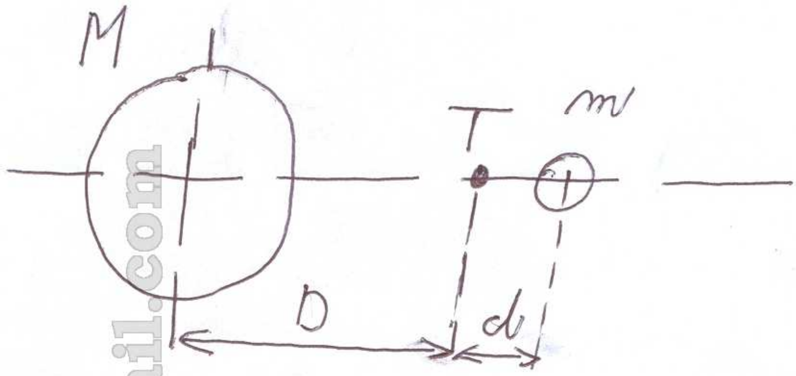
$$\boxed{F = \mathcal{K}Mm\left(\frac{1}{d^2} - \frac{1}{8(d-0{,}5R)}\right)}$$

V ktorom mieste na priamej spojnici medzi Zemou a Mesiacom sa intenzita spoločného gravitačného poľa rovná nule, Keď hmotnosť Mesiaca je **1/81** hmotnosti Zeme.

$$m = \frac{1}{81} M$$

m - MESIAC

M - ZEM



$$K_1 = \varkappa \frac{M}{D^2} \qquad K_2 = \varkappa \frac{m}{d^2}$$

$$m = \frac{1}{81} M$$

$$|\vec{K_1}| = |\vec{K_2}|$$

$$\cancel{\varkappa} \frac{\cancel{M}}{D^2} = \cancel{\varkappa} \frac{\cancel{M}}{81 d^2}$$

$$\frac{1}{D^2} = \frac{1}{81 d^2} \quad / \circlearrowleft$$

$$D^2 = 81 d^2 \quad / \sqrt{\ }$$

$$\boxed{D = 9d}$$

Vyjadrite závislosť zrýchlenia voľného pádu g_h od výšky **h** nad povrchom Zeme! V akej výške **h** zrýchlenie voľného pádu g_h tvorí 0,25 zrýchlenia voľného pádu g_0 na povrchu Zeme ?

$$g_h = f(h)$$

$$g_h = 0{,}25\, g_0$$

$$R_0 = 6370\,km = 6370 \cdot 10^3\,m$$

$$h = ?$$

$$g_0 = \varkappa \frac{M}{R^2} \Rightarrow \varkappa = \frac{g_0 R^2}{M}$$

$$g_h = \varkappa \frac{M}{(h+R)^2} = \frac{g_0 R^2}{\cancel{M}} \cdot \frac{\cancel{M}}{(h+R)^2}$$

$$g_h = g_0 \left(\frac{R}{h+R}\right)^2$$

$$g_h = 0{,}25\, g_0$$

$$0{,}25\, \cancel{g_0} = \cancel{g_0} \left(\frac{R}{h+R}\right)^2 \Big/ \sqrt{}$$

$$\sqrt{0{,}25} = \frac{R}{h+R} \quad \Big/ \cdot (h+R)$$

$$\sqrt{0{,}25}\,(h+R) = R$$

$$\sqrt{0{,}25}\,h + \sqrt{0{,}25}\,R = R$$

$$h = \frac{R - \sqrt{0{,}25}\,R}{\sqrt{0{,}25}} = \frac{R\cancel{(1-\sqrt{0{,}25})}}{\cancel{\sqrt{0{,}25}}} = R = \underline{\underline{6370\,km}}$$

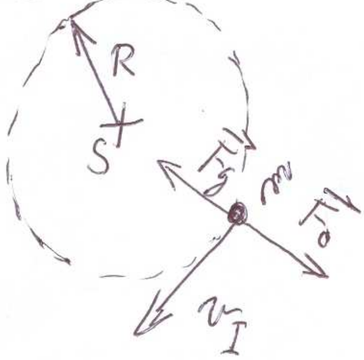
$$\boxed{\begin{array}{l} 1-\sqrt{0{,}25} = 1-0{,}5 = 0{,}5 \\ \sqrt{0{,}25} = 0{,}5 \end{array}}$$

Určte 1. kozmickú rýchlosť t.j. vodorovnú rýchlosť, ktorú treba udeliť telesu na povrchu Zeme, aby sa pohybovalo ako umelá družica Zeme po kruhovej dráhe. (**R=6370 km**)

$$\underline{R = 6370\,km}$$

$$v_I = ?$$

$$|\vec{F}_o| = |\vec{F}_g|$$



$$\frac{v_I^2}{\cancel{R}}\cancel{m} = \varkappa \frac{M\cancel{m}}{R^{\cancel{2}}}$$

$$v_I^2 = \varkappa \frac{M}{R}$$

HMOTNOSŤ ZEME

$M = 5{,}96 \cdot 10^{24}\,kg$

$$v_I = \sqrt{\varkappa \frac{M}{R}}$$

$$v_I = \sqrt{6{,}67 \cdot 10^{-11} \frac{5{,}96 \cdot 10^{24}}{6370 \cdot 10^3}} = 7901\,m\,s^{-1}$$

Akú vodorovnú rýchlosť treba udeliť telesu vo výške **500 km** nad zemským povrchom ,aby sa pohybovalo ako umelá družica Zeme po kruhovej dráhe ak polomer Zeme je **R**. (**R=6370 km**)

$h = 500\,km = 500 \cdot 10^3\,m$

$R = 6370\,km = 6370 \cdot 10^3\,m$

$v = ?$

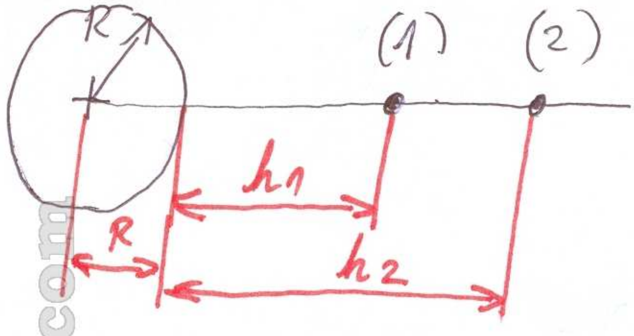
$$v = \sqrt{\varkappa \frac{M}{R+h}} =$$

$$= \sqrt{6{,}67 \cdot 10^{-11} \frac{5{,}96 \cdot 10^{24}}{6370 \cdot 10^3 + 500 \cdot 10^3}} = 7606\,m\,s^{-1}$$

Dve družice sa pohybujú okolo Zeme po kruhových dráhach vo výškach h_1 , h_2 od povrchu. Vyjadrite pomer rýchlostí $\mathbf{v_1/v_2}$ a pomer obežných dôb $\mathbf{T_1/T_2}$ týchto družíc, keď polomer Zeme je **R**.

$$h_1, h_2$$

$$\frac{v_1}{v_2} = ? \qquad \frac{T_1}{T_2} = ?$$



$$\frac{v_1^2}{\cancel{R+h_1}} = \varkappa \frac{M}{(R+h_1)^{\cancel{2}}}$$

$$\frac{v_2^2}{\cancel{R+h_2}} = \varkappa \frac{M}{(R+h_2)^{\cancel{2}}}$$

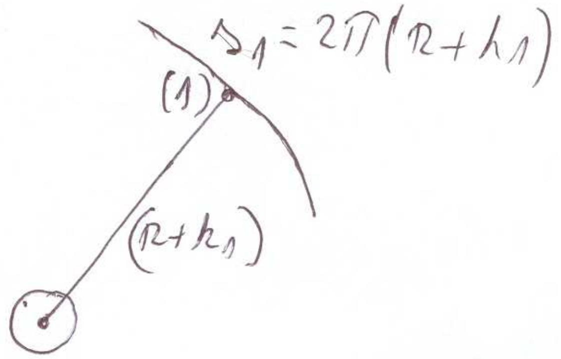
$$\frac{v_1^2}{v_2^2} = \frac{\cancel{\varkappa} \frac{\cancel{M}}{R+h_1}}{\cancel{\varkappa} \frac{\cancel{M}}{R+h_2}} = \frac{R+h_2}{R+h_1} \quad \Big/ \sqrt{}$$

$$\boxed{\frac{v_1}{v_2} = \sqrt{\frac{R+h_2}{R+h_1}}}$$

$$T_1 = \frac{s_1}{v_1} \qquad T_2 = \frac{s_2}{v_2}$$

$$s_1 = 2\pi(R+h_1)$$

$$s_2 = 2\pi(R+h_2)$$



$$v_1 = \sqrt{\varkappa\frac{M}{(R+h_1)}}$$

$$v_2 = \sqrt{\varkappa\frac{M}{(R+h_2)}}$$

$$T_1 = \frac{s_1}{v_1} = \frac{2\pi(R+h_1)}{\sqrt{\frac{\varkappa M}{R+h_1}}}$$

$$T_2 = \frac{s_2}{v_2} = \frac{2\pi(R+h_2)}{\sqrt{\frac{\varkappa M}{R+h_2}}}$$

$$\frac{T_1}{T_2} = \frac{\dfrac{2\pi(R+h_1)}{\sqrt{\frac{\varkappa M}{R+h_1}}}}{\dfrac{2\pi(R+h_2)}{\sqrt{\frac{\varkappa M}{R+h_2}}}} = \frac{(R+h_1)\sqrt{\frac{\varkappa M}{R+h_1}}}{(R+h_2)\sqrt{\frac{\varkappa M}{R+h_2}}} =$$

$$= \sqrt{\frac{(R+h_1)^2 \cancel{\frac{\mathcal{K} M}{R+h_1}}}{(R+h_2)^2 \cancel{\frac{\mathcal{K} M}{R+h_2}}}} = \boxed{\sqrt{\frac{R+h_1}{R+h_2}} = \frac{T_1}{T_2}}$$

$$\frac{T_1}{T_2} = \sqrt{\frac{R+h_1}{R+h_2}} \qquad \frac{v_1}{v_2} = \sqrt{\frac{R+h_2}{R+h_1}}$$

$$\boxed{\frac{T_1}{T_2} = \frac{v_2}{v_1}}$$

!

V akej výške **h** musí byť umelá družica Zeme, aby sa nachádzala nad tým istým miestom zemského povrchu. (Predpokladáme , že družica sa pohybuje po dráhe tvaru kružnice) $g_o = 9{,}81 ms^{-1}$, **R=6370 km**.

$T = 1 deň$

$h = ?$

$T = 24h \cdot 3600 s = 86400 s$

$$T = \frac{2\pi(R+h)}{\sqrt{\frac{\varkappa M}{R+h}}} = \frac{2\pi d}{\sqrt{\frac{\varkappa M}{d}}} \Big/ ^2$$

$R + h = d$

$$T^2 = \frac{4\pi^2 d^2}{\frac{\varkappa M}{d}}$$

$$\frac{T^2 \varkappa M}{d} = 4\pi^2 d^2 \Big/ \cdot d$$

$$T^2 \varkappa M = 4\pi^2 d^3 \Big/ \frac{1}{4\pi^2}$$

$$\frac{T^2 \varkappa M}{4\pi^2} = d^3$$

$$\sqrt[3]{\frac{T^2 \varkappa M}{4\pi^2}} = d$$

$$h = d - R = \sqrt[3]{\frac{T^2 \varkappa M}{4\pi^2}} - R =$$

$$= \sqrt[3]{\frac{(86400)^2 \cdot 6{,}67 \cdot 10^{-11} \cdot 5{,}96 \cdot 10^{24}}{4\pi^2}} - 637 \cdot 10^4 =$$

$$= 35833319{,}72\,m = \underline{\underline{35833{,}3\,km}}$$

Vypočítajte hmotnosť Slnka, keď poznáte hodnotu konštanty
k = 6,67.10^{-11} Nm^{2}kg^{-2} a vzdialenosť Zeme od Slnka , ktorá je **150.10^6 km**

$$\varkappa = 6{,}67 \cdot 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$$

$$d = 150 \cdot 10^6\,\mathrm{km} = 150 \cdot 10^9\,\mathrm{m}$$

$$F_g = \varkappa \frac{Mm}{d^2}$$

M - HMOTNOSŤ SLNKA

m - HMOTNOSŤ ZEME

$$F_o = \frac{v^2}{d} m$$

$$s = v \cdot t$$

$$2\pi d = v \cdot (365 \cdot 24 \cdot 3600)$$

$\nearrow \varphi$

$$v = \frac{2\pi d}{\varphi}$$

$$|\vec{F_g}| = |\vec{F_o}|$$

$$\varkappa \frac{M \cancel{m}}{d^{\cancel{2}}} = \frac{v^2}{\cancel{d}} \cancel{m}$$

$$\frac{\varkappa M}{d} = \left(\frac{2\pi d}{\varphi}\right)^2 \Rightarrow M = d \left(\frac{2\pi d}{\varphi}\right)^2 \frac{1}{\varkappa} =$$

$$= 150 \cdot 10^9 \left(\frac{2\pi \cdot 150 \cdot 10^9}{365 \cdot 24 \cdot 3600}\right)^2 \cdot \frac{1}{6{,}67 \cdot 10^{-11}} = 2 \cdot 10^{30}\,\mathrm{kg}$$

9. Elektrické pole

Akú hodnotu by musela mať gravitačná konštanta κ, aby vodíkový atóm mohol fungovať na báze gravitačných účinkov.

$m_e = 9{,}1.10^{-31}$kg , $m_p = 1{,}67.10^{-27}$kg , $e = 1{,}602.10^{-19}$ As , $\varepsilon_o = 8{,}854.10^{-12}\ m^{-3}kg^{-1}s^4A^2$

$m_e = 9{,}1.10^{-31}\ kg$

$m_p = 1{,}67.10^{-27}\ kg$

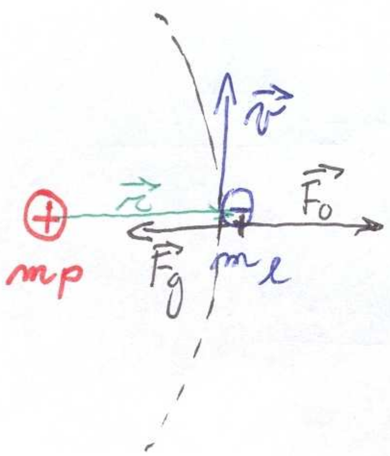
$e = 1{,}602.10^{-19}\ C \qquad e_p = e_e = e$

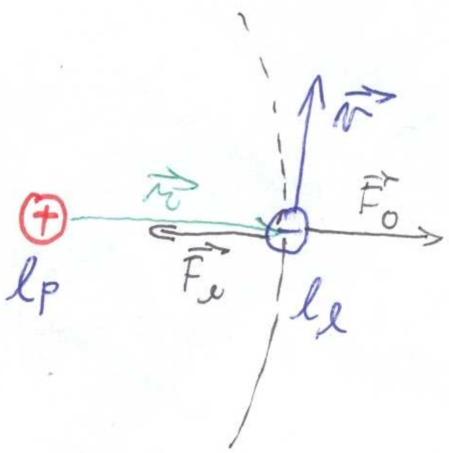
$\varepsilon_0 = 8{,}854.10^{-12}\ m^{-3}kg^{-1}s^4A^2$

$\kappa' = ?$

$$\frac{v^2}{r} = \kappa' \frac{m_e m_p}{r^2} \qquad\qquad \frac{v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$|\vec{F}_o| = |-\vec{F}_g| \qquad\qquad |\vec{F}_o| = |-\vec{F}_e|$$





$$\varkappa' \frac{m_e \cdot m_p}{\cancel{r^2}} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{\cancel{r^2}}$$

$$\varkappa' = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{m_e m_p} = \frac{1}{4\pi\, 8{,}854 \cdot 10^{-12}} \cdot \frac{(1{,}602 \cdot 10^{-19})^2}{9{,}1 \cdot 10^{-31} \cdot 1{,}67 \cdot 10^{-27}}$$

$$\varkappa' = 1{,}51 \cdot 10^{29}$$

Dve rovnako veľké guľôčky majú eklektické náboje $Q_1 = 24.10^{-6}$C a $Q_2 = -18.10^{-6}$C . Akou silou pôsobia na seba vo vzdialenosti 6cm vo vákuu ? Akou silou budú na seba pôsobiť v tej istej vzdialenosti, ak sa predtým uviedli do vzájomného styku ?
$\varepsilon_0 = 8{,}854.10^{-12}\ m^{-3}kg^{-1}s^4A^2$

$Q_1 = 24 \cdot 10^{-6} C$

$Q_2 = -18 \cdot 10^{-6} C$

$r = 6\,cm = 6 \cdot 10^{-2}\,m$

$F_1 = ?\quad F_2 = ?$

$$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{|Q_1||Q_2|}{r^2} = \frac{1}{4\pi\, 8{,}854 \cdot 10^{-12}} \cdot \frac{24 \cdot 10^{-6} \cdot 18 \cdot 10^{-6}}{(6 \cdot 10^{-2})^2}$$

$$F_1 = 1{,}78 \cdot 10^{3}\,N$$

$Q_1 = 24 \cdot 10^{-6}\,C$

$Q_2 = -18 \cdot 10^{-6}\,C$

$\boxed{\text{KONTAKT}}$

$$Q = Q_1 = Q_2 = \frac{Q_1 + Q_2}{2} = \frac{24 \cdot 10^{-6} + (-18 \cdot 10^{-6})}{2}$$

$$Q = 3 \cdot 10^{-6}\,C$$

$$F_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q^2}{r^2} = \frac{1}{4\pi\, 8{,}854 \cdot 10^{-12}} \cdot \frac{(3 \cdot 10^{-6})^2}{(6 \cdot 10^{-2})^2}$$

$$F_2 = \underline{22{,}46\,N}$$

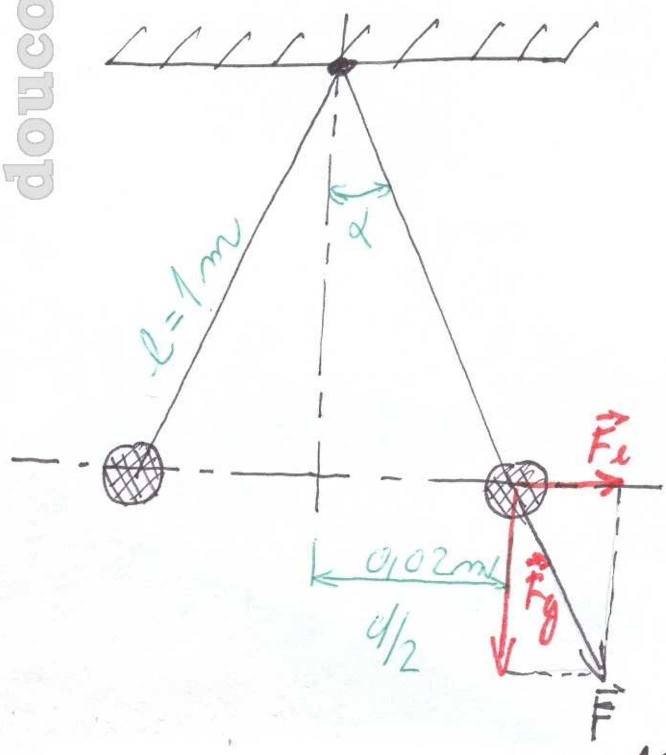
Dve elektricky rovnako nabité guľôčky, každá hmotnosti 5.10^{-4} kg boli vo vákuu zavesené v jednom bode na dvoch nitiach 1m dlhých a odpudzovaním sa od seba vzdialili na vzdialenosť 4cm. Aký veľký náboj je na každej z nich ? $\varepsilon_0 = 8{,}854.10^{-12}\ m^{-3}kg^{-1}s^4A^2$

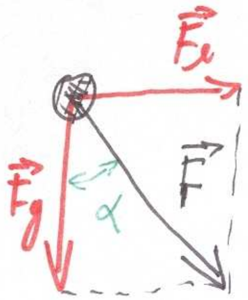
$m_1 = m_2 = 5 \cdot 10^{-4}\,kg$

$l = 1\,m$

$d = 4\,cm = 4 \cdot 10^{-2}\,m$

$Q_1 = Q_2 = ?$





$$\frac{\frac{d}{2}}{l} = \sin\alpha = 0{,}02$$

$$\alpha = \arcsin(0{,}02) = 1{,}14^\circ$$

$$\frac{F_e}{F_g} = \operatorname{tg}\alpha$$

$$F_e = F_g \operatorname{tg}\alpha = m g \operatorname{tg}\alpha = 5 \cdot 10^{-4} \cdot 9{,}8 \cdot \operatorname{tg} 1{,}14$$

$$F_e = 9{,}7 \cdot 10^{-5}\,N$$

$$F_e = \frac{Q^2}{r^2}\frac{1}{4\pi\varepsilon_0} \Rightarrow Q = \sqrt{F_e r^2 4\pi\varepsilon_0}$$

$$Q = \sqrt{9{,}7 \cdot 10^{-5} \cdot (0{,}04)^2 \cdot 4\pi \cdot 8{,}85 \cdot 10^{-12}}$$

$$Q = 4{,}15 \cdot 10^{-9}\,C$$

Na nitiach rovnakej dĺžky sú zavesené nabité guľôčky hmotnosti m = 1g a polomeru R = 3 mm. V dôsledku Coulombovských síl sa guľôčky rozostúpia o uhol α. Keď ich ponoríme do kvapaliny s ε_r = 1,1 uhol sa zachová. Aká je jej hustota ?

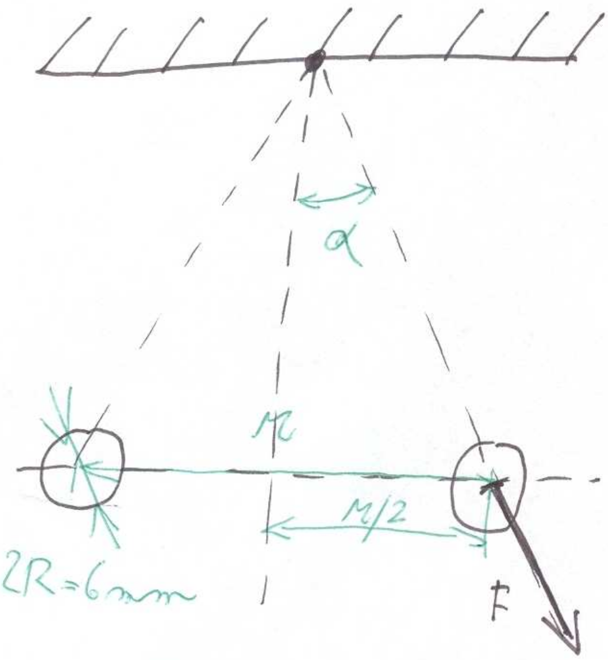
$m_1 = m_2 = 1g = 1 \cdot 10^{-3} kg$

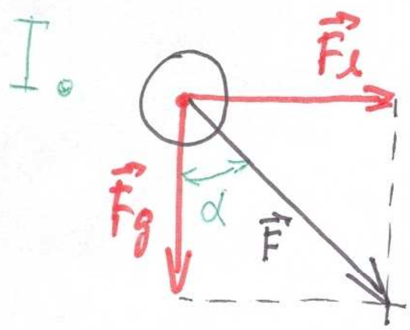
$R = 3mm = 3 \cdot 10^{-3} m$

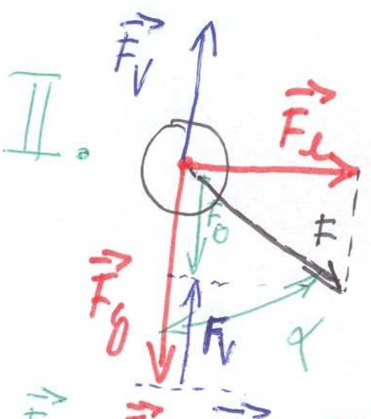
$\varepsilon_r = 1{,}1$

$\alpha_1 = \alpha_2$

$\rho = ?$







$\vec{F_o} = \vec{F_g} - \vec{F_V}$!

$\frac{F_e}{F_g} = \operatorname{tg} \alpha \qquad F_g = mg \qquad F_e = F_g \operatorname{tg} \alpha$

$F_e = F_g \operatorname{tg} \alpha = \boxed{g m \operatorname{tg} \alpha = \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{r^2}}$ I.

$F_o = F_g - F_V$

$$F_0 \, \mathrm{tg}\,\alpha = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{Q^2}{r^2}$$

$$F_V = V\rho \cdot g = \frac{4}{3}\pi R^3 \rho g$$

$$\boxed{\Big(\underbrace{gm}_{F_g} - \underbrace{\tfrac{4}{3}\pi R^3 \rho g}_{F_V}\Big)\,\mathrm{tg}\,\alpha = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\,\frac{Q^2}{r^2}} \quad \mathrm{II.}$$

I. $\Rightarrow$

$$\frac{1}{r^2} = \frac{(gm\,\mathrm{tg}\,\alpha)(4\pi\varepsilon_0)}{Q^2}$$

II. $\Rightarrow$

$$\frac{1}{r^2} = \frac{\left[(gm\,\mathrm{tg}\,\alpha) - \left(\frac{4}{3}\pi R^3 \rho g\,\mathrm{tg}\,\alpha\right)\right](4\pi\varepsilon_0\varepsilon_r)}{Q^2}$$

I. $\Leftrightarrow$ II.

$$\frac{(\cancel{g}m\,\cancel{\mathrm{tg}\,\alpha})\cancel{(4\pi\varepsilon_0)}}{\cancel{Q^2}} = \frac{\left[(\cancel{g}m\,\cancel{\mathrm{tg}\,\alpha}) - \left(\frac{4}{3}\pi R^3 \rho \cancel{g}\,\cancel{\mathrm{tg}\,\alpha}\right)\right](\cancel{4\pi\varepsilon_0}\varepsilon_r)}{\cancel{Q^2}}$$

$$m = \left[m - \left(\frac{4}{3}\pi R^3 \rho\right)\right]\varepsilon_r$$

$$m = \varepsilon_r m - \frac{4}{3}\pi R^3 \rho \varepsilon_r$$

$$m - \varepsilon_r m = -\frac{4}{3}\pi R^3 \rho \varepsilon_r$$

$$\rho = \frac{m(1-\varepsilon_r)}{-\frac{4}{3}\pi R^3 \varepsilon_r} = \frac{1 \cdot 10^{-3}(1-1{,}1)}{-\frac{4}{3}\pi \cdot (3 \cdot 10^{-3})^3 \cdot 1{,}1}$$

$$\rho = 806{,}45 \; kg\,m^{-3}$$

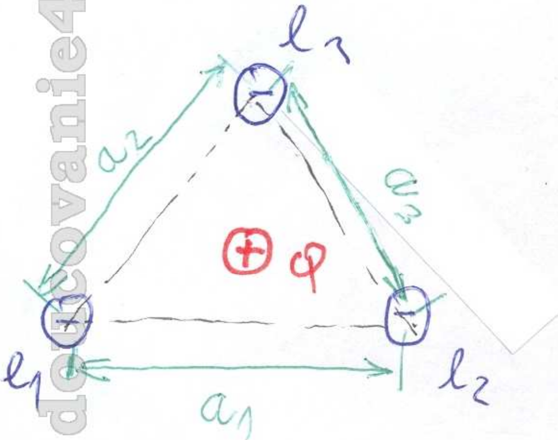
V rohoch rovnostranného trojuholníka sú umiestnené bodové náboje veľkosti e, Aký veľký bodový náboj treba umiestniť do ťažiska trojuholníka, aby boli náboje v rovnováhe ?

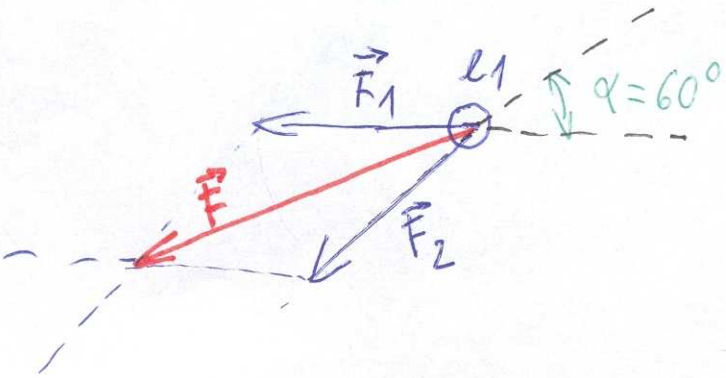
$e_1 = e_2 = e_3 = e$

$a_1 = a_2 = a_3 = a$

$F_1 = F_2 = F_0$

$$\frac{F_2}{\sin 30^\circ} = \frac{F_1}{\sin 30^\circ} = \frac{F}{\sin 120^\circ}$$

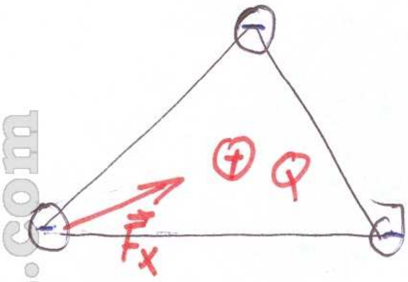




$$F = \sin 120^\circ \frac{F_0}{\sin 30^\circ} = \frac{\sqrt{3}}{2} \frac{F_0}{\frac{1}{2}} = \sqrt{3} F_0$$

$$|-\vec{F}_x| = |\vec{F}|$$

$$|-\vec{F}_x| = \sqrt{3} F_0$$



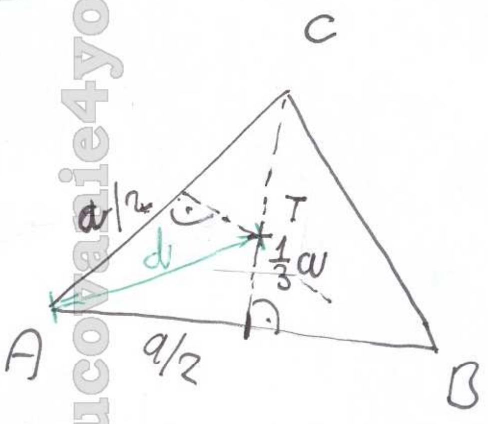
$$F_0 = \frac{1}{4\pi\varepsilon_0} \frac{l^2}{a^2}$$

$$\sqrt{3} \frac{1}{\cancel{4\pi\varepsilon_0}} \frac{l^{\cancel{2}}}{a^2} = \frac{\cancel{l} Q}{d^2} \cdot \frac{1}{\cancel{4\pi\varepsilon_0}}$$

$$\sqrt{3} \frac{l}{\cancel{a^2}} = \frac{Q}{\frac{13}{36} \cancel{a^2}}$$

$$\sqrt{3}\, l = \frac{36}{13} Q$$

$$Q = \sqrt{3} \frac{13}{36} l = \frac{1}{\sqrt{3}} l$$



$$d^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{3}\right)^2 = \frac{13}{36} a^2$$

Aké veľké je napätie medzi dvoma bodmi A a B , ktoré sú uložené vo vákuu v elektrostatickom poli náboja $Q = 5.10^{-7}$ C, a to tak, že bod A je od náboja Q vo vzdialenosti $r_1 = 2$cm a bod B vo vzdialenosti $r_2 = 10$cm v tom istom smere ? Akú prácu treba vykonať na prenesenie náboja $q = 3.10^{-8}$C z miesta B do miesta A.
$\varepsilon_o = 8{,}854.10^{-12}\ m^{-3}kg^{-1}s^4A^2$

A, B

$Q = 5.10^{-7} C$

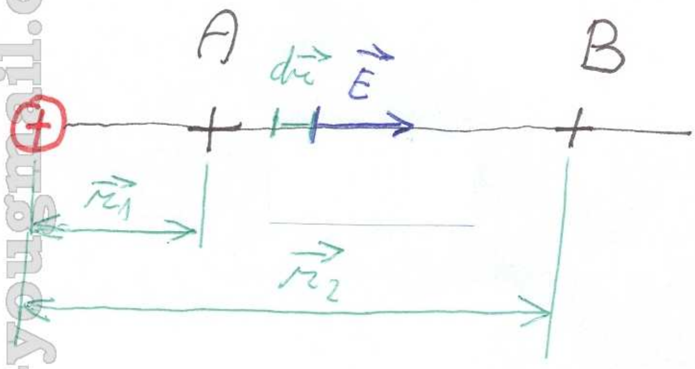
$r_1 = 2\,cm = 2.10^{-2}\,m$

$r_2 = 10\,cm = 10.10^{-2}\,m$

$q = 3.10^{-8} C$

$W_{AB} = ?$

$\varphi = ?$



$d\varphi = \vec{E}\,\vec{dr}$

$$\varphi = \int_{r_1}^{r_2} \vec{E}\,\vec{dr} = \int_{r_1}^{r_2} \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}} \cdot dr = \frac{Q}{4\pi\varepsilon_0}\left[-\frac{1}{r}\right]_{r_1}^{r_2} =$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{5.10^{-7}}{4\pi\varepsilon_0}\cdot\left(\frac{1}{2.10^{-2}} - \frac{1}{10.10^{-2}}\right) =$$

$$= \underline{179{,}75.10^{3}\,V}$$

Z vodivej mydlovej bubliny polomeru $R_1 = 2$ cm nabitej na potenciál $\varphi_1 = 10^4$V vznikne pri prasknutí kvapka vody polomeru $R_2 = 0{,}05$cm. Aký veľký je potenciál kvapky ?

$R_1 = 2\ cm = 2.10^{-2}\ m$

$\varphi_1 = 10^4\ V$

$R_2 = 0{,}05\ cm = 5.10^{-4}\ m$

$\varphi_2 = ?$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}$$

$$\varphi_1 = \lim_{c\to\infty}\int_{R_1}^{c}\underbrace{\frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}}_{\vec{E}}\ \underbrace{dx}_{\cdot\ \vec{dr}} = \frac{Q}{4\pi\varepsilon_0}\lim_{c\to\infty}\left[-\frac{1}{x}\right]_{R_1}^{c} =$$

$$= \frac{Q}{4\pi\varepsilon_0}\lim_{c\to\infty}\left(\frac{1}{R_1} - \underbrace{\cancel{\frac{1}{c}}}_{=0}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{R_1} = \varphi_1$$

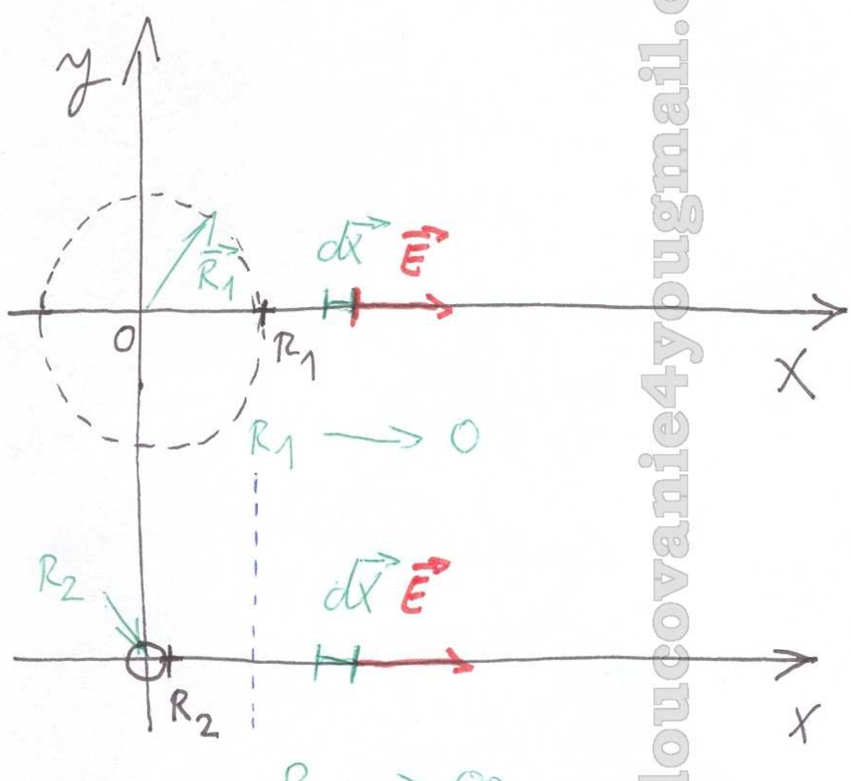
$\varphi_1 = 10^4\ V$

$$\frac{Q}{4\pi\varepsilon_0}\frac{1}{R_1} = 10^4$$

$$Q = 10^4 \cdot 4\pi\varepsilon_0 \cdot R_1 = 2{,}22 \cdot 10^{-8}\,C$$

$$\varphi_2 = \frac{1}{R_2} Q \frac{1}{4\pi\varepsilon_0} = \frac{1}{5 \cdot 10^{-4}} \cdot 2{,}22 \cdot 10^{-8} \cdot \frac{1}{4\pi\varepsilon_0}$$

$$\varphi_2 = 399{,}047 \cdot 10^3\,V = \underline{\underline{400\,kV}}$$



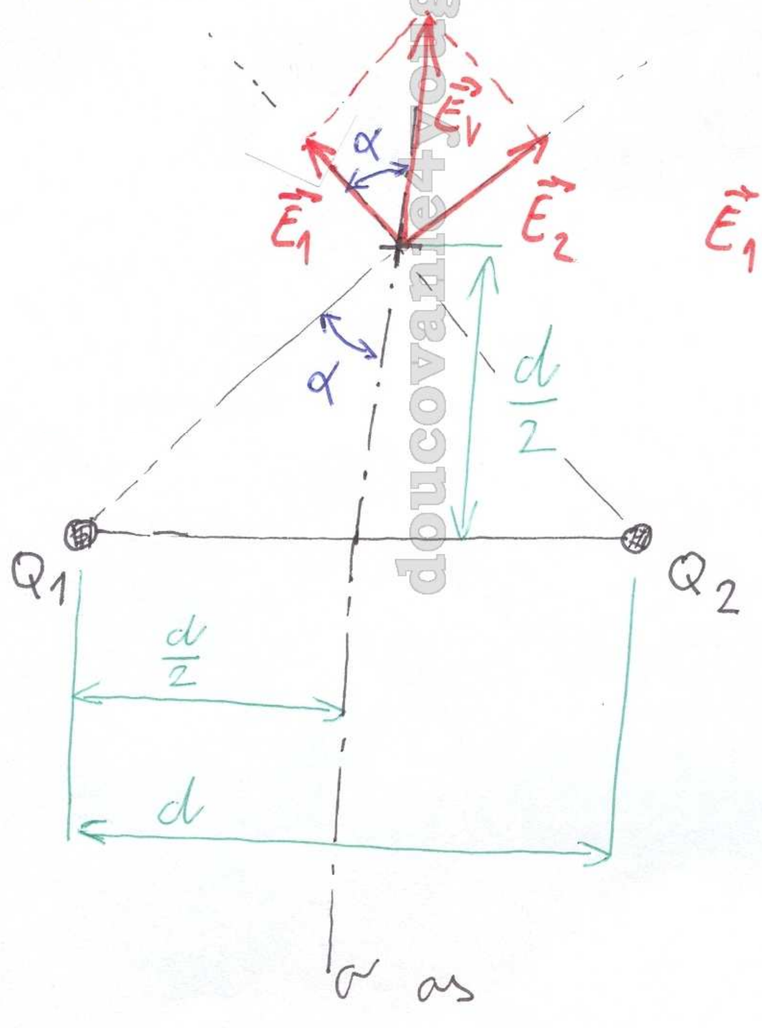
Dva rovnaké bodové náboje veľkosti 1μC sú vo vzdialenosti d=1m. Vpočítajte intenzitu elektrostatického poľa na osi spojnice vo vzdialenosti d/2 od jej stredu !
$\varepsilon_0 = 8{,}854.10^{-12}\ m^{-3}kg^{-1}s^4A^2$

$Q = 1\mu C = 1.10^{-6} C$

$d = 1m$

$E_V = ?$

$$a^2 = \left(\frac{d}{2}\right)^2 + \left(\frac{d}{2}\right)^2 = \frac{d^2}{4} + \frac{d^2}{4} = \frac{1}{2}d^2$$



$\vec{E_1} = \vec{E_2}$

$$|\vec{E}| = |\vec{E}_1| = |\vec{E}_2| = \frac{1}{\cancel{4}_2 \pi \varepsilon_0} \frac{Q}{\frac{1}{\cancel{2}} d^2} = \frac{1}{2\pi\varepsilon_0} \frac{Q}{d^2}$$

$$\frac{\frac{d}{2}}{\frac{d}{2}} = \operatorname{tg}\alpha \qquad \alpha = \operatorname{arctg} 1 = 45^\circ$$

$$\frac{|\vec{E}|}{|\vec{E}_V|} = \sin\alpha$$

$$|\vec{E}_V| = \frac{|\vec{E}|}{\sin\alpha} = \frac{1}{\sin\alpha} \cdot \frac{1}{2\pi\varepsilon_0} \cdot \frac{Q}{d^2} =$$

$$= \frac{1}{\sin 45^\circ} \cdot \frac{1}{2\pi\varepsilon_0} \cdot \frac{(1 \cdot 10^{-6})^2}{1^2} = \underline{\underline{2{,}54 \cdot 10^4 \, V\,m^{-1}}}$$

10. Jednosmerný elektrický prúd

Akou veľkou rýchlosťou sa pohybujú elektróny v medenom $^{64}_{29}Cu$ drôte s prierezom 1mm^2 keď ním prechádza prúd 6A ?
(M_{Cu}= 63,54 .10 $^{-3}$ kg mol^{-1} , ρ = 8,9.10^3 kg.m^{-3} , N_A = 6,022.10^{23}mol^{-1} , e=1,602.10^{-19} As)

$S = 1 mm^2 = 1 \cdot 10^{-6} m^2$

$I = 6A$

$M_{cu} = 63,54 \cdot 10^{-3} kg\ mol^{-1}$

$\rho = 8,9 \cdot 10^{3} kg\ m^{-3}$

$N_A = 6,022 \cdot 10^{23} mol^{-1}$

$e = 1,602 \cdot 10^{-19} As$

$v = ?$

$$dI = \frac{d}{dt}(e n_0 v\, dS\, dt)$$

$dt = 0$

$$I = e n_0 v S$$

$$v = \frac{I}{e n_0 S}$$

n_0 – POČET VOĽNÝCH ELEKRÓNOV V $1m^3$ KOVU

n_0 = POČTU ATÓMOV V $1 m^3$ KOVU → PREDPOKLAD

$$m = \rho \cdot V = 8.9 \cdot 10^3 \cdot 1 = 8{,}9 \cdot 10^3 \, kg$$

$$N = \frac{m}{M_{Cu}} = \frac{8{,}9 \cdot 10^3}{63{,}54 \cdot 10^{-3}} = 141 \cdot 10^3 \, mol^{-1}$$

$$n_0 = N_A \cdot N = 141 \cdot 10^3 \cdot 6{,}022 \cdot 10^{23} = 8{,}5 \cdot 10^{28} \, m^{-3}$$

$$v = \frac{I}{e n_0 S} = \frac{6}{1{,}602 \cdot 10^{-19} \cdot 8{,}5 \cdot 10^{28} \cdot 1 \cdot 10^{-6}} = 4{,}4 \cdot 10^{-4} \, m\,s^{-1}$$

Hustota prúdu v hliníkovom vodiči je **j**=1Amm^{-2} . Vypočítajte rýchlosť v usporiadaného pohybu elektrónov za predpokladu, že počet voľných elektrónov v 1cm^3 sa rovná počtu atómov v tomto objeme !

$$j = 1 A\,mm^{-2} = 1 \cdot 10^6 A\,m^{-2}$$

$$\rho = 2{,}7 \cdot 10^3 \, kg\,m^{-3}$$

$$e = 1{,}602 \cdot 10^{-19} \, A\,s$$

$$N_A = 6{,}022 \cdot 10^{23} \, mol^{-1}$$

$$v = ?$$

$$j = \frac{I}{S}$$

$$v = \frac{I}{S} \frac{1}{en} = j \frac{1}{en}$$

$m = \rho \cdot V$

$$N = \frac{m}{M_{Al}} = \frac{\rho V}{M_{Al}}$$

$$n_0 = N_A \cdot N = N_A \frac{\rho \cancel{V}^{=1}}{M_{Al}} = \frac{N_A \rho}{M_{Al}} =$$

$$= \frac{6{,}022 \cdot 10^{23} \cdot 2{,}7 \cdot 10^{3}}{27 \cdot 10^{-3}} = 6{,}022 \cdot 10^{28}\ m^{-3}$$

$$v = j \frac{1}{en} = 1 \cdot 10^{6} \frac{1}{1{,}602 \cdot 10^{-19} \cdot 6{,}022 \cdot 10^{28}}$$

$$v = \underline{\underline{1{,}03 \cdot 10^{-4}\ m\,s^{-1}}}$$

**Hustota prúdu v medenom vodiči je j=3Amm^{-2} . Aká je intenzita elektrického poľa vo vodiči ?
(ρCu = 17.10^{-9} Ωm)**

$j = 3A\,mm^{-2} = 3 \cdot 10^{6} A\,m^{-2}$

$\rho_{cu} = 17 \cdot 10^{-9}\ \Omega m$

$U = ?$

$U = R \cdot I \quad / \cdot \frac{1}{S}$

$$\frac{U}{S} = R\frac{I}{S} = R \cdot j \qquad R = \rho\frac{l}{S}$$

$$\frac{U}{\cancel{S}} = \rho\frac{l}{\cancel{S}}\, j$$

$$U = \rho\, l\, j$$

AK $l = 1m$

$$E = \rho \cdot j = 17 \cdot 10^{-9} \cdot 3 \cdot 10^{6} = 5{,}1 \cdot 10^{-2}\ Vm^{-1}$$

Aké množstvo elektrického náboja Q prejde vodičom za čas to = 10s , keď:
A, Prúd je stály I= 5A ?
B, Prúd rovnomerne rastie od 0A do 3A

$t_0 = 0$

$t_1 = 10s$

a) $I = konšt = 5A$

b) $I = f(t)\ (0-3)A$

$Q = ?$

a)

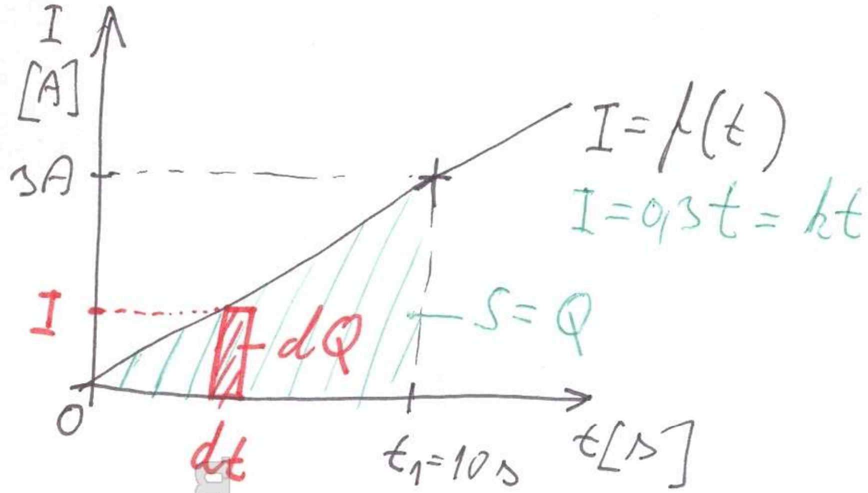
$$Q = I \cdot t_0 = 5A \cdot 10s = 50C$$

$$\boxed{dQ = I\,dt}$$

$$I = kt$$

$$\boxed{I = 0{,}3t}$$

$$k = \frac{3}{10} = 0{,}3$$



$$dQ = \underbrace{0{,}3t}_{I}\,dt \;\Big/\int$$

$$Q = \int_{t_0}^{t_1} 0{,}3t\,dt = 0{,}3\left[\frac{t^2}{2}\right]_0^{10} = 0{,}3\left[\frac{10^2}{2}\right] = \underline{\underline{15\,C}}$$

Vypočítajte náboj Q, ktorý prejde obvodom s odporom R=5Ω pri rovnomernom narastaní napätia z hodnoty U_1 =3V na U_2=7V za čas t=2s !

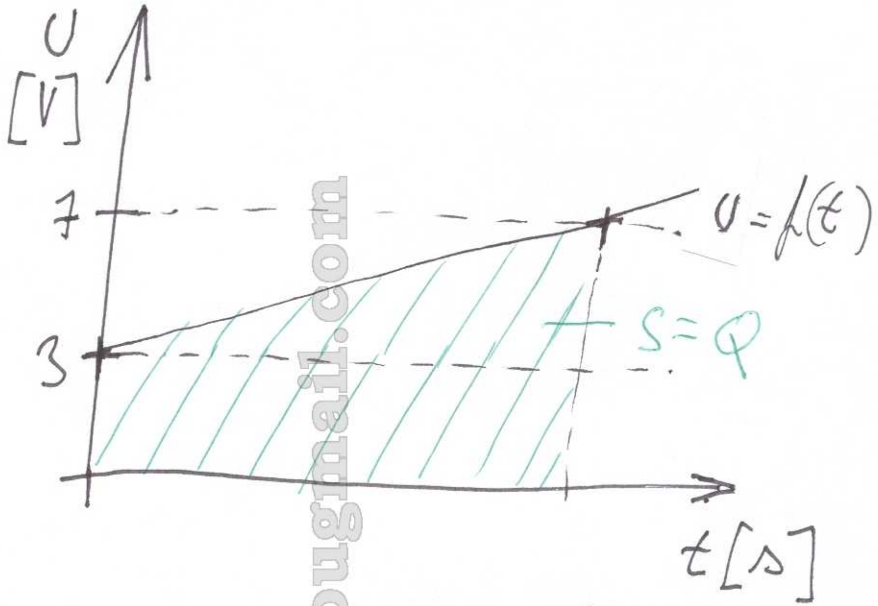
$R = 5\,\Omega$

$U_1 = 3\,V$

$U_2 = 7\,V$

$t = 2\,s$

$Q = ?$



$$I = \frac{U}{R}$$

$$U = 3 + kt$$

$$7 = 3 + 2k$$

$$k = 2$$

$$\boxed{U = 3 + 2t} \quad U = f(t)$$

$$I = \frac{1}{R}(3 + 2t)$$

$$dQ = I\,dt = \frac{1}{R}(3+2t)\,dt \;\Big/ \int$$

$$Q = \int_{t_1}^{t_2} \frac{1}{R}(3+2t)\,dt = \int_{t_1}^{t_2} \left(\frac{3}{R} + \frac{2t}{R}\right) dt =$$

$$= \left[\frac{3t}{R} + \frac{2t^2}{2R}\right]_0^2 =$$

$$= \left(\frac{3.2}{5} + \frac{2.2^2}{2.5}\right) - \left(\frac{3.0}{5} - \frac{2.0^2}{2.5}\right) = 2C$$

Aký je pomer tiaži medi a hliníka, keď z nich vyrobíme vodiče rovnakej dĺžky a odporu ? **($\rho Cu = 17.10^{-9}\ \Omega m$, $\rho Al = 0{,}029.10^{-6}\ \Omega m$, $\rho Cu' = 8{,}9.10^3\ kgm^{-3}$, $\rho Al' = 2{,}7.10^3\ kgm^{-3}$)**

$\rho_{cu} = 17.10^{-9}\ \Omega m$

$\rho_{Al} = 0{,}029.10^{-6}\ \Omega m$

$\rho'_{cu} = 8{,}9.10^3\ kg\ m^{-3}$

$\rho'_{Al} = 2{,}7.10^3\ kg\ m^{-3}$

$\frac{G_{cu}}{G_{Al}} = ?$

$R_{Al} = R_{cu}$

$l_1 = l_2$

$R = \rho \frac{l}{S}$

$R_{Al} = \rho_{Al} \frac{l_1}{S_1} \Rightarrow S_1 = \frac{\rho_{Al} . l}{R_{Al}}$

$R_{cu} = \rho_{cu} \frac{l_2}{S_2} \Rightarrow S_2 = \frac{\rho_{cu} . l}{R_{cu}}$

$m = \rho' . V$

$$G_{Al} = \rho'_{Al} \cdot V_1 \, g$$

$$G_{cu} = \rho'_{cu} V_2 \, g$$

$$G_{Al} = \rho'_{Al} \left(\frac{\rho_{Al} \, l_1}{R_{Al}} \right) l_1 \, g$$

$$G_{cu} = \rho'_{cu} \left(\frac{\rho_{cu} \cdot l_2}{R_{cu}} \right) l_2 \, g$$

$$\frac{G_{cu}}{G_{Al}} = \frac{\rho'_{cu} \left(\frac{\rho_{cu} \cancel{l_2}}{\cancel{R_{cu}}} \right) \cancel{l_2 \, g}}{\rho'_{Al} \left(\frac{\rho_{Al} \cdot \cancel{l_1}}{\cancel{R_{Al}}} \right) \cancel{l_1 \, g}}$$

$$\frac{G_{cu}}{G_{Al}} = \frac{\rho'_{cu} \cdot \rho_{cu}}{\rho'_{Al} \cdot \rho_{Al}} = \frac{8{,}9 \cdot 10^3 \cdot 17 \cdot 10^{-9}}{2{,}7 \cdot 10^3 \cdot 0{,}029 \cdot 10^{-6}}$$

$$\frac{G_{cu}}{G_{Al}} = 1{,}93$$

Aký má byť prierez medeného vedenia, aby v prípade 1km dlhého vedenia a 1A-ového prúdu nebolo napätie väčšie ako 20 V ! $\rho Cu = 17.10^{-9}\ \Omega m$

$l = 1km = 1000\ m$

$I = 1A$

$U \leq 20V$

$\rho_{cu} = 0{,}017 \cdot 10^{-6}\ \Omega m$

$S \geq ?$

$R = \rho_{cu} \dfrac{l}{S}$ $\qquad I = \dfrac{U}{R} \Rightarrow U = R \cdot I$

$RI \leq 20$

$\rho_{cu} \dfrac{l}{S} \cdot I \leq 20$

$\rho_{cu} \dfrac{l}{20} I \leq S$

$\dfrac{0{,}017 \cdot 10^{-6} \cdot 1000 \cdot 1}{20} \leq S$

$8{,}5 \cdot 10^{-4}\ m^2 \leq S$

$0{,}85\ mm^2 \leq S$

Elektrický obvod sa skladá z troch vodičov rovnakej dĺžky zhotovených z rovnakého materiálu, ktoré sú zapojené za sebou. Prierezy vodičov sú: $S_1 = 1\,mm^2$ $S_2 = 2\,mm^2$ $S_3 = 3\,mm^2$.
Rozdiel potenciálov na koncoch obvodu je U= 12V. Učte úbytok napätia na každom vodiči.

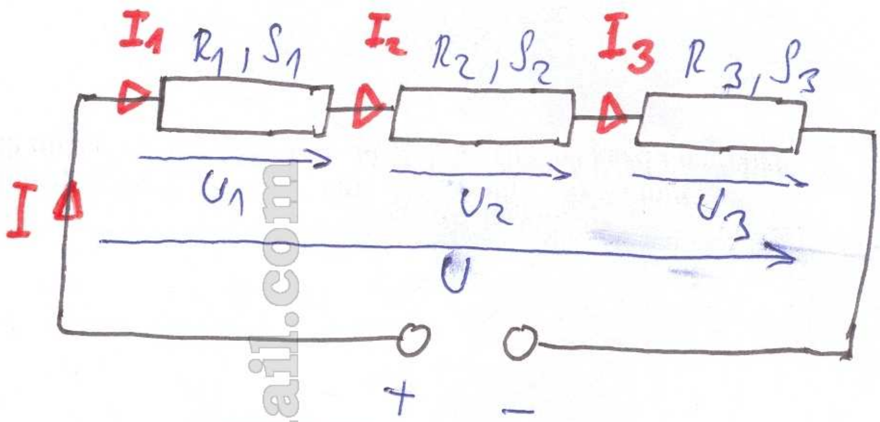
$S_1 = 1\,mm^2$

$S_2 = 2\,mm^2$

$S_3 = 3\,mm^2$

$U = 12\,V$

$U_1, U_2, U_3 = ?$



$$S_1 < S_2 < S_3$$

$$R_1 > R_2 > R_3$$

$$U_1 > U_2 > U_3$$

$$\boxed{R = \rho \frac{l}{S}}$$

$$\boxed{U = R \cdot I}$$

I. KZ

$$I_1 = I_2 = I_3 = I$$

II. KZ

$$U = U_1 + U_2 + U_3$$

$$R = R_1 + R_2 + R_3$$

$$I = \frac{U}{R} = \frac{U}{R_1 + R_2 + R_3}$$

$$R = \underbrace{\rho \frac{l}{S_1}}_{R_1} + \underbrace{\rho \frac{l}{S_2}}_{R_2} + \underbrace{\rho \frac{l}{S_3}}_{R_3} = \rho l \left(\frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} \right)$$

$$I = \frac{U}{R} = \frac{U}{\rho l \left(\frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} \right)}$$

$$U_1 = R_1 I \qquad U_2 = R_2 \cdot I \qquad U_3 = R_3 I$$

$$R_1 = \rho \frac{l}{S_1} \qquad R_2 = \rho \frac{l}{S_2} \qquad R_3 = \rho \frac{l}{S_3}$$

$$U_1 = R_1 \cdot I = \cancel{\rho} \frac{\cancel{l}}{S_1} \cdot \frac{U}{\cancel{\rho l} \left(\frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} \right)} =$$

$$= \frac{U}{\frac{S_1}{S_1} + \frac{S_1}{S_2} + \frac{S_1}{S_3}} = \frac{12}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3}} = \underline{\underline{6{,}54\,V}}$$

TAK ISTO AKO U_1 AJ U_2 A U_3 POTOM:

$$U_2 = \frac{U}{\frac{S_2}{S_1} + \frac{S_2}{S_2} + \frac{S_2}{S_3}} = \frac{12}{\frac{2}{1} + \frac{2}{2} + \frac{2}{3}} = \underline{\underline{3{,}27\,V}}$$

$$U_3 = \frac{U}{\frac{S_3}{S_1} + \frac{S_3}{S_2} + \frac{S_3}{S_3}} = \frac{12}{\frac{3}{1} + \frac{3}{2} + \frac{3}{3}} = \underline{\underline{2{,}18\,V}}$$

11. Stacionárne magnetické pole

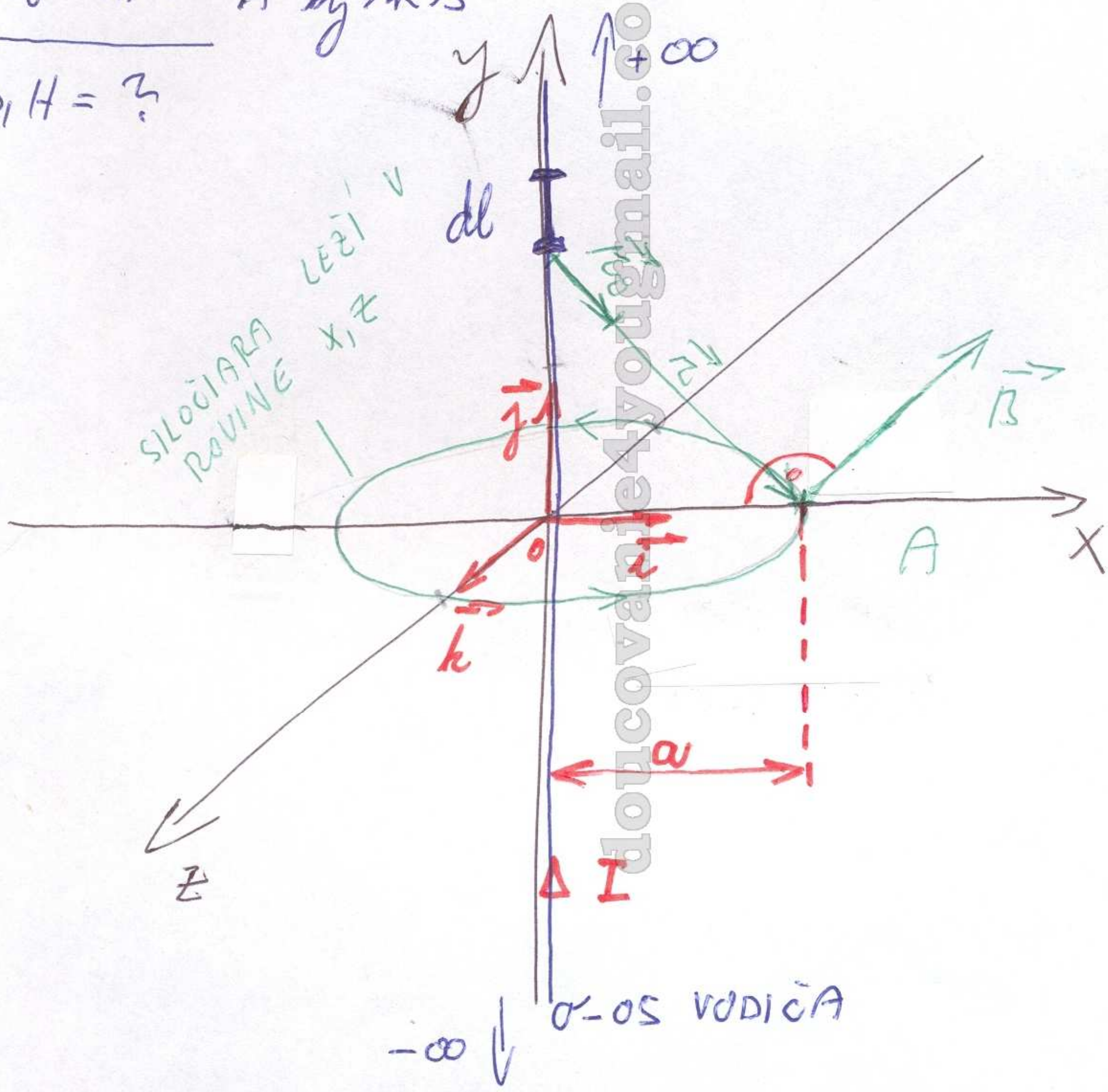
Určíte indukciu a intenzitu magnetického poľa vo vzdialenosti a = 5cm od veľmi dlhého priameho vodiča, keď ním preteká prúd I=5A. ($\mu_o= 4\pi.10^{-7}\ A^{-2}kg\ m\ s^{2}$)

$a = 5cm = 5.10^{-2} m$

$I = 5A$

$\mu_0 = 4\pi.10^{-7} A^{-2} kg\ m\ s^2$

$B, H = ?$



$\boxed{I \parallel \sigma \parallel \vec{j} \parallel dl \parallel y}$ $\boxed{\vec{B} \perp \vec{i}}$ $\boxed{\vec{B} \parallel \vec{k}}$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \vec{r_0}}{r^2} = \frac{\mu_0 I}{4\pi} \cdot \frac{|\vec{dl}| \cdot |\vec{r_0}| \cdot \sin\alpha}{r^2}$$

$$\vec{A} \times \vec{B} = |\vec{A}| \cdot |\vec{B}| \sin\alpha$$

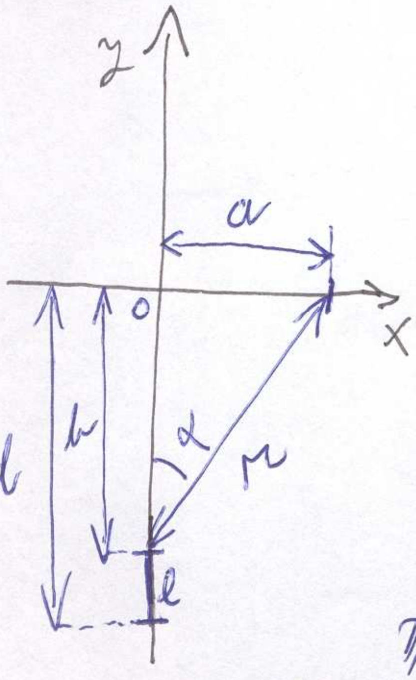
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{dl \sin\alpha}{r^2}$$

$$\vec{dl} \to d\alpha$$

$$r^2 \to \alpha, a$$

$$l = d - b = d - a \cot g\,\alpha$$

$$\boxed{\begin{array}{l} \frac{b}{a} = \cot g\,\alpha \\ b = a \cot g\,\alpha \end{array}}$$



$$l = d - a \cot g\,\alpha \,\Big|'$$

$$dl = -\frac{a}{-\sin^2\alpha} d\alpha = \boxed{\frac{a}{\sin^2\alpha} d\alpha}$$

$$\boxed{\frac{a^2}{\sin^2\alpha} = r^2} \qquad \frac{a}{r} = \sin\alpha$$

$$B = 2 \int_0^{\pi/2} \frac{\mu_0 I}{4\pi} \frac{\sin\alpha}{r^2} dl =$$

$$B = 2\int_0^{\pi/2} \frac{\mu_0 I}{4\pi} \frac{1}{\underbrace{\boxed{\frac{a^2}{\sin^2\alpha}}}_{r^2}} \cdot \sin\alpha \underbrace{\boxed{\frac{a}{\sin^2\alpha}\, d\alpha}}_{dl} =$$

$$= \frac{\cancel{2}\mu_0 I \cancel{a}}{\cancel{4}\pi \cancel{a^2}} \int_0^{\pi/2} \cancel{\sin^2}\alpha \sin\alpha \frac{1}{\cancel{\sin^2}\alpha}\, d\alpha =$$

$$= \frac{\mu_0 I}{2\pi a} \int_0^{\pi/2} \sin\alpha\, d\alpha = \frac{\mu_0 I}{2\pi a} \left[-\cos\alpha\right]_0^{\pi/2} =$$

$$= \frac{\mu_0 I}{2\pi a} \left(-\cos\frac{\pi}{2} - (-\cos 0)\right) = \frac{\mu_0 I}{2\pi a} \left(-0 - (-1)\right)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi a}} = \frac{4\pi \cdot 10^{-7} \cdot 5}{2\pi \cdot 5 \cdot 10^{-2}} = \underline{\underline{2 \cdot 10^{-5}\, T}}$$

$$H = \frac{B}{\mu} = \frac{2 \cdot 10^{-5}}{4\pi \cdot 10^{-7}} = \underline{\underline{15{,}9\, A\, m^{-1}}}$$

Vypočítajte intenzitu magnetického poľa vyvolaného úsekom priameho vodiča, ktorým preteká prúd I=10A, a to v bode nachádzajúcom sa vo vzdialenosti d= 5cm kolmo od stredu tohto úseku vodiča ! Dĺžka vodiča je taká, že ju vidieť z bodu , v ktorom intenzitu magnetického poľa počítame, pod zorným uhlom 60^0. Prostredie v okolí vodiča je vákuum.

$I = 10A$

$a = 5cm = 5 \cdot 10^{-2} m$

$\beta = 60^\circ$

$H = ?$

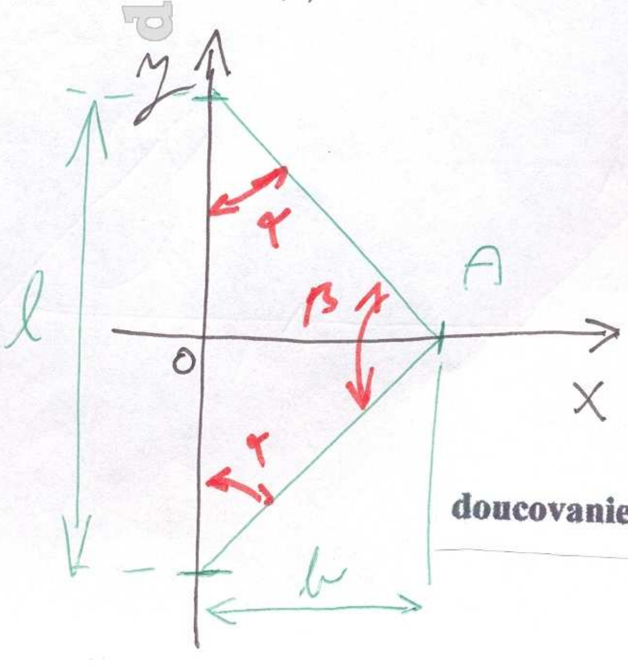
$$H = \frac{B}{\mu_0}$$

$$H = \frac{1}{\mu_0} \frac{\mu_0 I}{2\pi a} \int_{(1)}^{(2)} \sin\alpha \, d\alpha = \boxed{\frac{I}{2\pi a} \int_{(1)}^{(2)} \sin\alpha \, d\alpha}$$

AKO PREDOŠLÝ PRÍKLAD LEN TREBA URČIŤ (1)(2)

$\beta = 60^\circ = \frac{1}{3}\pi$

$\alpha = \frac{180 - \beta}{2} = 60^\circ$



$$H = \frac{I}{2\pi a}\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \sin\alpha \, d\alpha = \frac{I}{2\pi a}\Big[-\cos\alpha\Big]_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} =$$

$$= \frac{I}{2\pi a}\left(-0-\left(-\frac{1}{2}\right)\right) = \boxed{\frac{I}{4\pi a} = H}$$

$$H = \frac{I}{4\pi a} = \frac{10A}{4\pi \cdot 5 \cdot 10^{-2} m} = 15{,}9 \, A\, m^{-1}$$

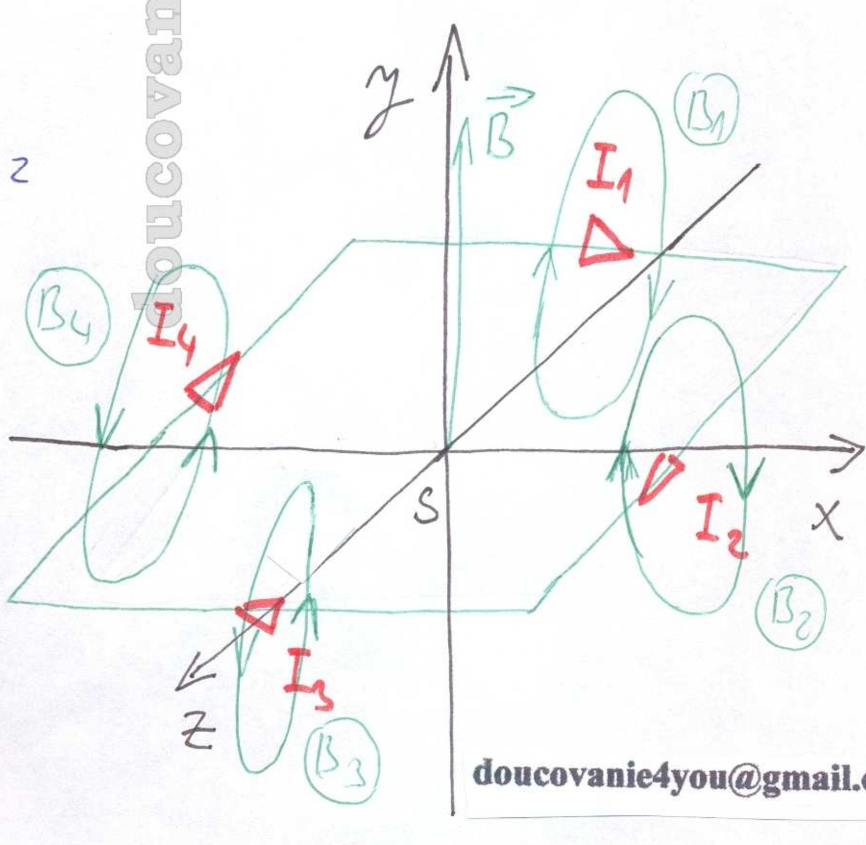
Vypočítajte magnetickú indukciu v strede závitu tvaru štvorca so stranou a= 10cm, ktorým preteká prúd I = 5A ! (μ_o= $4\pi.10^{-7}$ A^{-2}kg m s^{2})

$a = 10\, cm = 0{,}1\, m$

$I = 5\, A$

$\mu_0 = 4\pi \cdot 10^{-7} A^{-2} kg\, m\, s^{-2}$

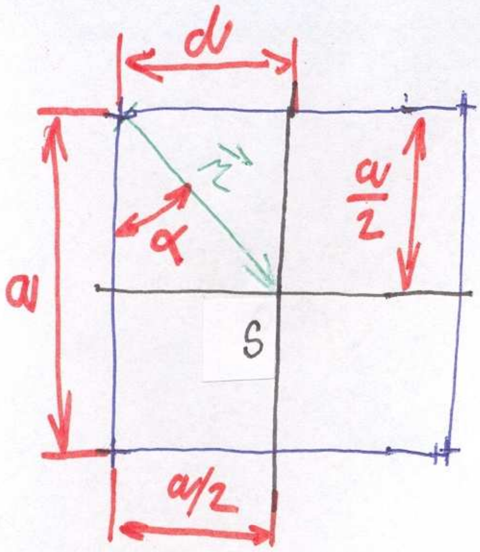
$B = ?$



$\vec{B} = \vec{B_1} + \vec{B_2} + \vec{B_3} + \vec{B_4}$

$\vec{B_1} \parallel \vec{B_2} \parallel \vec{B_3} \parallel \vec{B_4}$

$\vec{B_1} = \vec{B_2} = \vec{B_3} = \vec{B_4}$



$$\sin\alpha = \frac{\frac{a}{2}}{r} = \frac{a}{2r}$$

$$\cos\alpha = \frac{\frac{a}{2}}{r} = \frac{a}{2r}$$

$$\sin\alpha = \cos\alpha = \qquad \alpha = \frac{1}{4}\pi = 45^\circ$$

$$\sin\frac{1}{4}\pi = \cos\frac{1}{4}\pi = \frac{\sqrt{2}}{2}$$

$$d = \frac{a}{2}$$

$$B = \frac{\mu_0 I}{2\pi d}\cdot\boxed{4}\cdot\int_{1/4\pi}^{1/2\pi}\sin\alpha\, d\alpha = \frac{2\mu_0 I}{\pi d}\Big[-\cos\alpha\Big]_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} =$$

$$= \frac{2\mu_0 I}{\pi\frac{a}{2}}\left(-0-\left(-\frac{\sqrt{2}}{2}\right)\right) = \boxed{\frac{2\sqrt{2}\,\mu_0 I}{\pi\cdot a} = B}$$

$$B = \frac{2\cdot\sqrt{2}\cdot 4\pi\cdot 10^{-7}\cdot 5}{\pi\cdot 0{,}1} = 5{,}65\cdot 10^{-5}\,\mathrm{T}$$

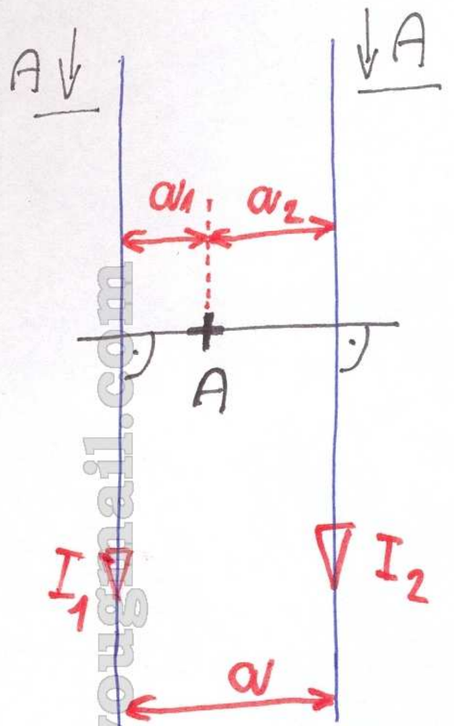
Vypočítajte indukciu magnetického poľa budeného dvoma priamymi , nekonečne dlhými rovnobežnými vodičmi, vzdialenými od seba a=10cm, ktorými prechádza ten istý prúd I=2A v tom istom smere, vo vzdialenosti a_1=4cm od prvého na spoločnej kolmej spojnici oboch vodičov! ($\mu_o = 4\pi.10^{-7}\ A^{-2} kg\ m\ s^2$)

$a = 10\ cm = 0{,}1\ m$

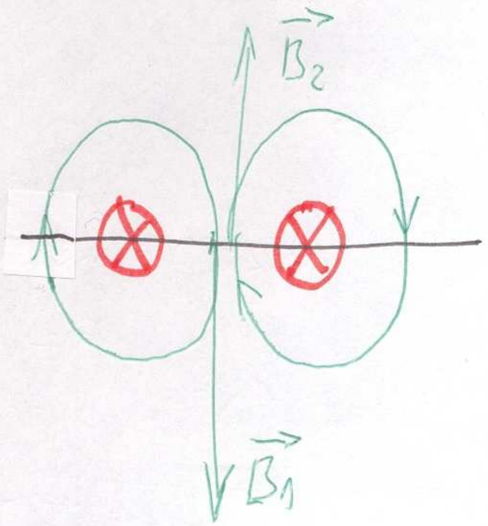
$a_1 = 4\ cm = 0{,}04\ m$

$I = 2A$

$B = ?\ [T]$



A-A



$I_1 = I_2 = I = 2A$

$a_2 = a - a_1 = 10 - 4 = 6\ cm = 0{,}06\ m$

$\vec{B} = \vec{B_2} - \vec{B_1}$

$|\vec{B_2}| > |\vec{B_1}|$

$$\vec{B} = \vec{B}_2 - \vec{B}_1 = \frac{\mu_0 I}{2\pi a_1} - \frac{\mu_0 I}{2\pi a_2} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{a_1} - \frac{1}{a - a_1}\right) =$$

$$= \frac{\mu_0 I}{2\pi}\left(\frac{a - 2a_1}{a_1(a - a_1)}\right) = \frac{4\pi \cdot 10^{-7} \cdot 2}{2\pi} \cdot \left(\frac{0{,}1 - 0{,}08}{0{,}04(0{,}1 - 0{,}04)}\right) =$$

$$= 3{,}33 \cdot 10^{-6}\, T$$

Určite intenzitu a indukciu magnetického poľa v strede kruhového vodiča polomeru R=5cm, keď ním preteká prúd I = 5A. ($\mu_o = 4\pi.10^{-7}\ A^{-2}kg\ m\ s^{2}$)

$R = 5\,cm = 0{,}05\,m$

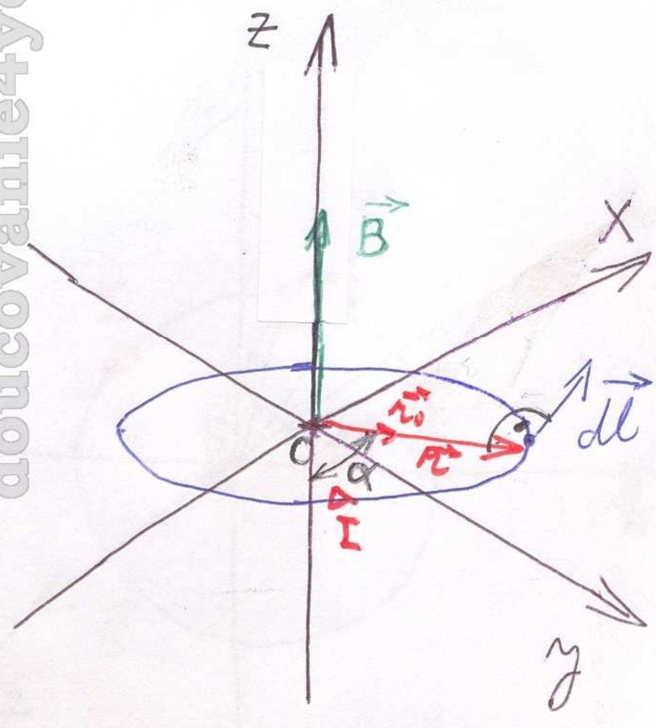
$I = 5A$

$\mu_0 = 4\pi \cdot 10^{-7} A^{-2} kg\, m\, s^{-2}$

$\vec{B}, \vec{H} = ?$

$\vec{B} \parallel$ s osou z

ZÁVIT LEŽÍ V ROVINE xy;



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \vec{r}_0}{r^2}$$

$$\vec{dl} \times \vec{r}_0 = |\vec{dl}| \cdot |\vec{r}_0| \cdot \sin\alpha$$

$$\vec{dl} \times \vec{r_0} = |\vec{dl}|\,|\vec{1}|.\sin 90^\circ = dl$$

$$dB = \frac{\mu_0 I}{4\pi}\frac{1}{r^2}\,dl$$

$$dH = \frac{I}{4\pi}\frac{1}{r^2}\,dl$$

$$B = \oint_0^{2\pi r} \frac{\mu_0}{4\pi}\frac{1}{r^2}\,dl = \frac{\mu_0}{4\pi}\frac{1}{r^2}\oint_0^{2\pi r} dl =$$

$$= \frac{\mu_0}{4\pi r^2}\Big[l\Big]_0^{2\pi r} = \frac{\mu_0}{4\pi r^2}\Big[2\pi r - 0\Big] = \frac{\mu_0\,\cancel{2\pi r}}{2\,\cancel{4\pi r^2}} =$$

$$= \boxed{\frac{\mu_0 I}{2r} = B}$$

$$= \frac{\mu_0 I}{2r} = \frac{4\pi.10^{-7}.5}{2\cdot 0{,}05} = \underline{\underline{6{,}25.10^{-5}\,T}}$$

$$H = \frac{B}{\mu} = \frac{6{,}25.10^{-5}}{4\pi.10^{-7}} = \underline{\underline{49{,}76\,A\,m^{-1}}}$$

Vodičom kruhového tvaru polomeru R = 10cm preteká prúd I = 2A. Vypočítajte indukciu magnetického poľa v mieste A na osi uvedeného vodiča vo vzdialenosti a= 10 cm od jeho stredu ! ($\mu_o = 4\pi.10^{-7}$ $A^{-2}kg\ m\ s^2$)

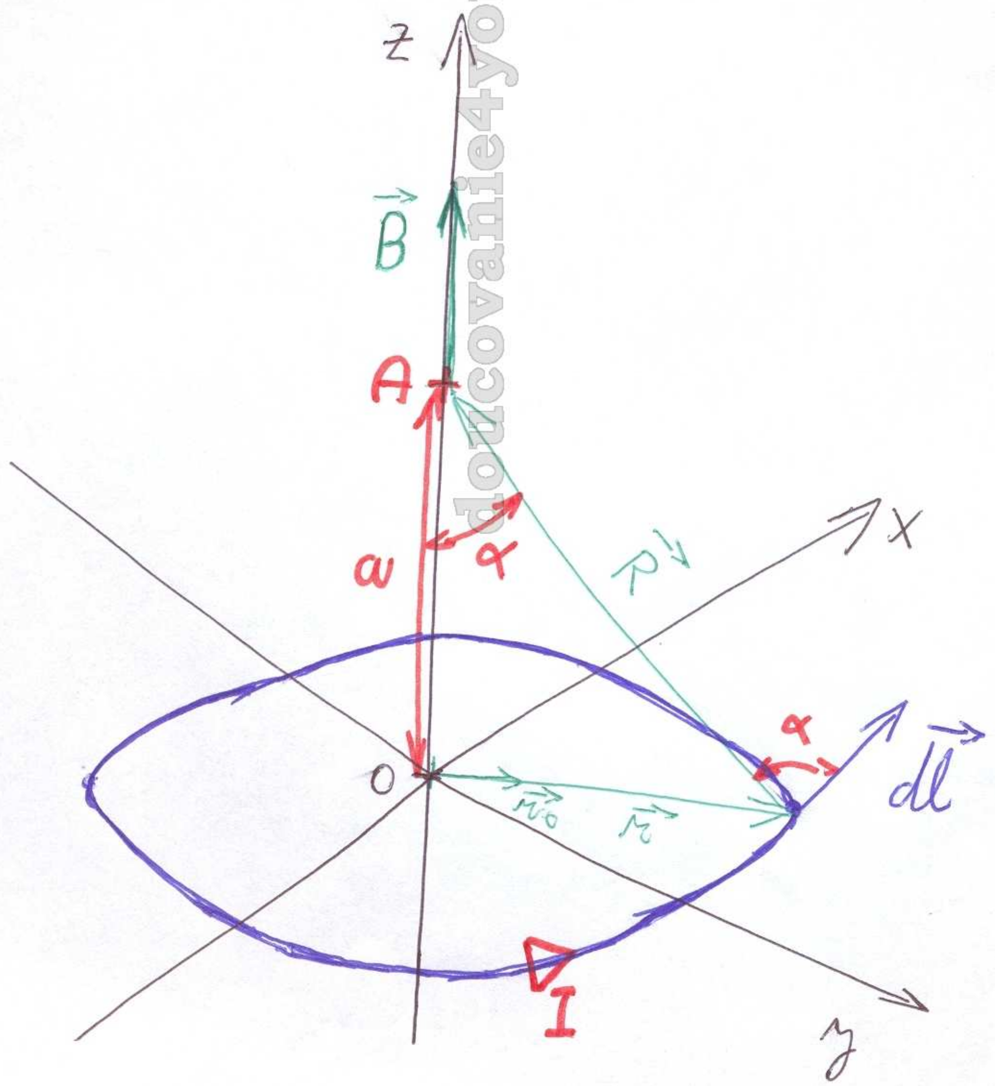
$r = 10\ cm = 0{,}1\ m$

$I = 2A$

$a = 10\ cm = 0{,}1\ m$

$\mu_0 = 4\pi \cdot 10^{-7} A^{-2} kg\ m\ s^{-2}$

$\vec{B} = ?\ [T]$



$$R^2 = r^2 + a^2 \Rightarrow R = \sqrt{r^2+a^2}$$

$$\frac{r}{R} = \sin\alpha = \frac{r}{\sqrt{r^2+a^2}}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \cdot \frac{|\vec{dl}||\vec{r_0}|\sin\alpha}{R^2} =$$

$$= \frac{\mu_0 I}{4\pi} \frac{|\vec{dl}||\vec{r_0}|}{R^2} \cdot \frac{r}{\sqrt{r^2+a^2}}$$

$$B = \oint_0^{2\pi r} \frac{\mu_0 I}{4\pi} \frac{1}{R^2} \frac{r}{\sqrt{r^2+a^2}} dl =$$

$$= \frac{\mu_0 I}{4\pi} \frac{1}{(r^2+a^2)} \cdot \frac{r}{\sqrt{r^2+a^2}} \oint_0^{2\pi r} dl =$$

$\frac{1}{R^2}$

$$= \frac{\mu_0 I}{4\pi} \frac{1}{(r^2+a^2)} \cdot \frac{r}{\sqrt{r^2+a^2}} \left[2\pi r - 0\right] =$$

$$= \frac{\mu_0 I}{\cancel{4\pi}_{\,2}} \frac{1}{r^2+a^2} \frac{r}{\sqrt{r^2+a^2}} (\cancel{2\pi} r) =$$

$$= \boxed{\frac{\mu_0 I r^2}{2\sqrt{(r^2+a^2)^3}} = B}$$

$$B = \frac{\mu_0 I r^2}{2\sqrt{(r^2+a^2)^3}} = \frac{4\pi \cdot 10^{-7} \cdot 2 \cdot 0{,}1}{2\sqrt{(0{,}1^2+0{,}1^2)^3}}$$

$$B = \underline{4{,}43 \cdot 10^{-5}\,T}$$

Dvoma vodičmi kruhového tvaru R_1= 10cm R_2= 15cm preteká prúd I_1= 2A, I_2= 5A tak že vo svojom okolí budia magnetické pole. Vypočítajte jeho intenzitu v bode A na osi týchto kruhových vodičov, ktorého vzdialenosti od ich stredov sú X_1= 5cm, X_2= 10cm ($\mu_o = 4\pi.10^{-7}\ A^{-2}kg\ m\ s^2$)

$I_1 = 2A$

$I_2 = 5A$

$R_1 = 10cm = 0{,}1m$

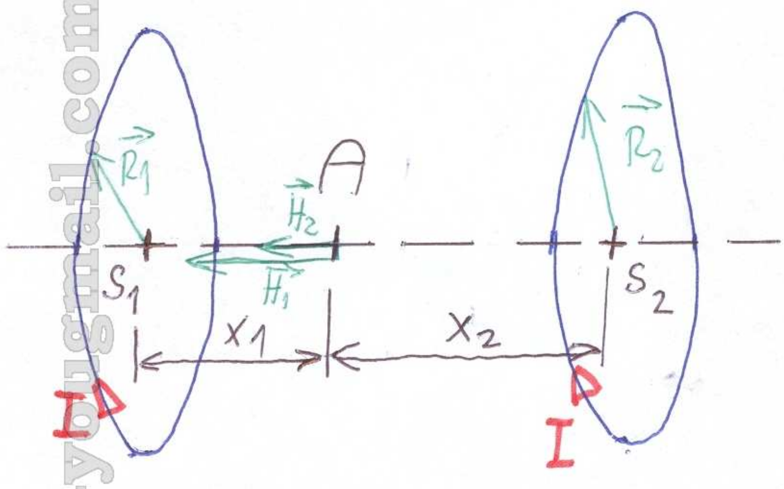
$R_2 = 15cm = 0{,}15m$

$X_1 = 5cm = 0{,}05m$

$X_2 = 10cm = 0{,}1m$

$\mu_0 = 4\pi \cdot 10^{-7} A^{-2} kg\ m\ s^{-2}$

$\vec{H} = ?\ (A m^{-1})$



$\vec{H_1} // \vec{H_2}$

$|\vec{H}| = |\vec{H_1}| + |\vec{H_2}|$

$$H = \frac{IR^2}{2\sqrt{(R^2+X^2)^3}}$$

$$H = \frac{I_1 R_1^2}{2\sqrt{(R_1^2+X_1^2)^3}} + \frac{I_2 R_2^2}{2\sqrt{(R_2^2+X_2^2)^3}} =$$

$$= \frac{2 \cdot 0{,}1^2}{2\,(0{,}1^2+0{,}05^2)^{\frac{3}{2}}} + \frac{5 \cdot 0{,}15^2}{2\,(0{,}15^2+0{,}1^2)^{\frac{3}{2}}} = \underline{16{,}53\ A m^{-1}}$$

Aký je polomer zakrivenia dráhy elektrónu s kinetickou energiou $E_k = 5keV$, ak sa pohybuje v homogénnom magnetickom poli s indukciou B = 0,005 T , kolmom na smer pohybu.
(m_e=3,1.10^{-31}kg 1eV=1,602.10^{-19}J e= 1,602.10^{-19} C)

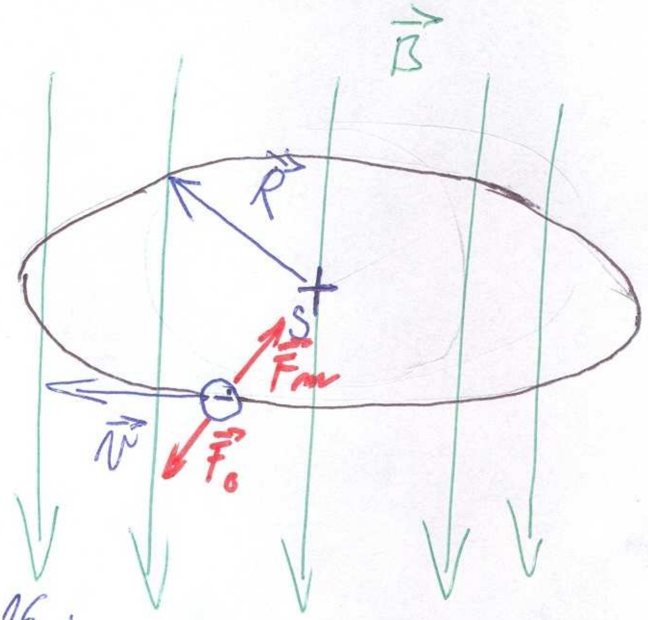
$E_k = 5 keV$

$B = 0,005 T$

$m_0 = 9,1 \cdot 10^{-31} kg$

$1eV = 1,602 \cdot 10^{-19} J$

$e = 1,602 \cdot 10^{-19} As = Q$



$R = ?$

$F = BQv$

$E = E_k \cdot 1,602 \cdot 10^{-19} = 8,01 \cdot 10^{-16} J$

$E = \frac{1}{2} m v^2$

$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{8,01 \cdot 10^{-16} \cdot 2}{9,1 \cdot 10^{-31}}} = 42 \cdot 10^6 m s^{-1}$

$F_0 = F_m$

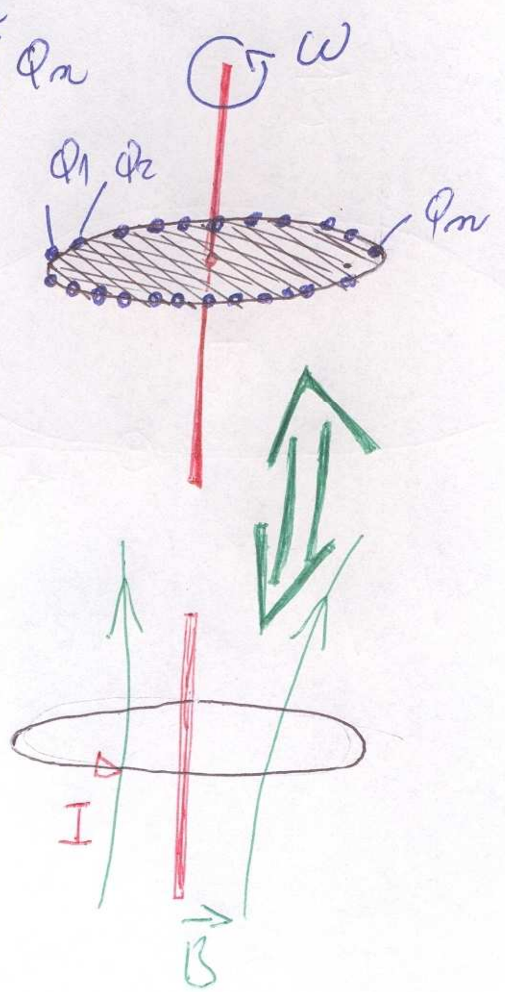
$m \frac{v^{\cancel{2}}}{R} = BQ\cancel{v}$

$R = \frac{vm}{BQ} = \frac{42 \cdot 10^6 \cdot 9,1 \cdot 10^{-31}}{0,005 \cdot 1,602 \cdot 10^{-19}} =$

$= \underline{\underline{0,047 m}}$

Na obvode kotúča polomeru R= 10 cm, je rovnomerne rozložený náboj $Q = 10^{-8}$C . Kotúč sa otáča okolo osi prechádzajúcej jeho stredom s frekvenciou $f = 100\ s^{-1}$. Vypočítajte aká veľká je magnetická indukcia v strede kotúča !

$$Q = \sum_{i=1}^{n} Q_n$$



$f = 100\ s^{-1}$

$Q = 1 \cdot 10^{-8}\ C$

$R = 10\ cm = 0{,}1\ m$

$$I = f \cdot Q = 100 \cdot 1 \cdot 10^{-8} = 1 \cdot 10^{-6}\ A$$

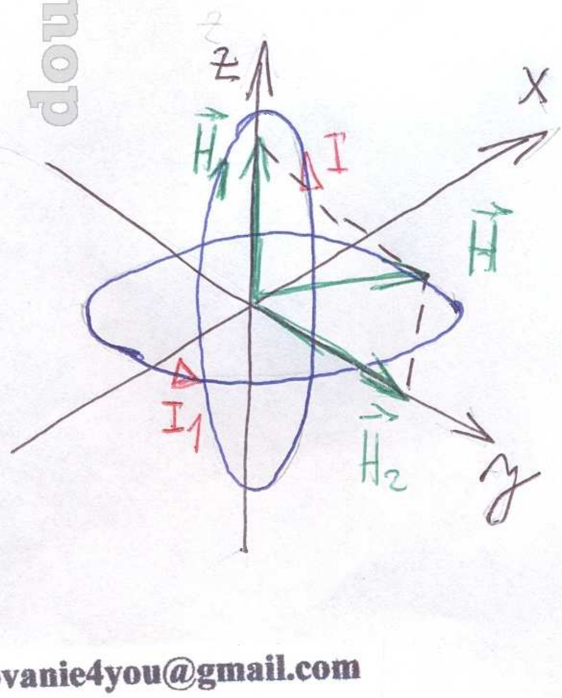
$$H = \frac{I}{2R} = \frac{1 \cdot 10^{-6}}{2 \cdot 0{,}1} = \underline{\underline{5 \cdot 10^{-6}\ Am^{-1}}}$$

Dva kruhové vodiče , každý polomeru R = 5cm , majú spoločný stred a ich roviny sú na seba kolmé. Vypočítajte smer a veľkosť intenzity magnetického poľa v strede závitov keď prúdy prechádzajúce vodičmi sú I_1=3A a I_2 = 4A !

$I_1 = 3A$

$I_2 = 4A$

$H = ?$



$$H_1 = \frac{I_1}{2R} = \frac{3}{2 \cdot 0{,}05} = 30\,A\,m^{-1}$$

$$H_2 = \frac{I_2}{2R} = \frac{4}{2 \cdot 0{,}05} = 40\,A\,m^{-1}$$

$$H = \sqrt{H_1^2 + H_2^2} = \sqrt{30^2 + 40^2} = 50\,A\,m^{-1}$$

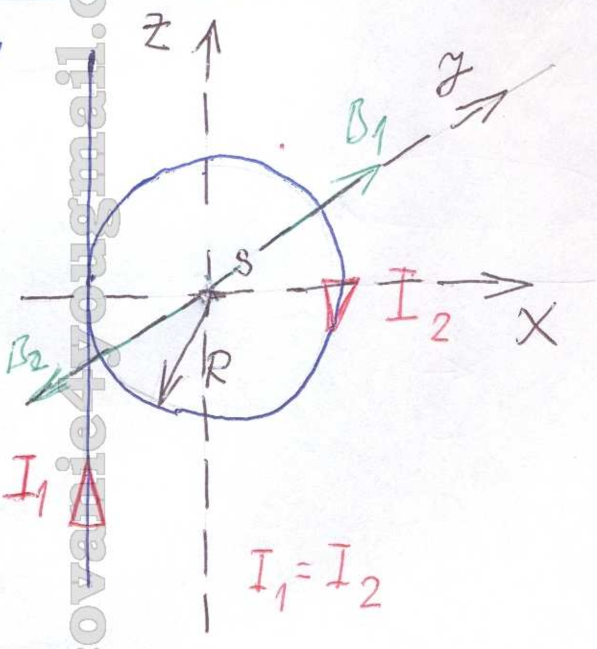
Veľmi dlhý priami vodič , ktorým tečie prúd I = 10A, vytvára v určitom mieste kruhový závit polomeru R = 4,28 cm ležiaci v rovine preloženej prúdovodičom. Vypočítajte veľkosť a smer magnetickej indukcie v strede uvedeného závitu.

$R = 4{,}28\,cm = 0{,}0428\,m = a$

$I = 10A$

$\mu_0 = 4\pi \cdot 10^{-7}\,A^{-2}\,kg\,m\,s^{-2}$

$B = ?$



$$B_1 = \frac{\mu_0 I}{2\pi a} = \frac{4\pi \cdot 10^{-7} \cdot 10}{2\pi \cdot 0{,}0428} = 4{,}67 \cdot 10^{-5}\,T$$

$$B_2 = \frac{\mu_0 I}{2R} = \frac{4\pi \cdot 10^{-7} \cdot 10}{2 \cdot 0{,}0428} = 1{,}46 \cdot 10^{-4}\,T$$

$$B = B_2 - B_1 = 1{,}46 \cdot 10^{-4} - 4{,}67 \cdot 10^{-5} = 1 \cdot 10^{-4}\,T$$

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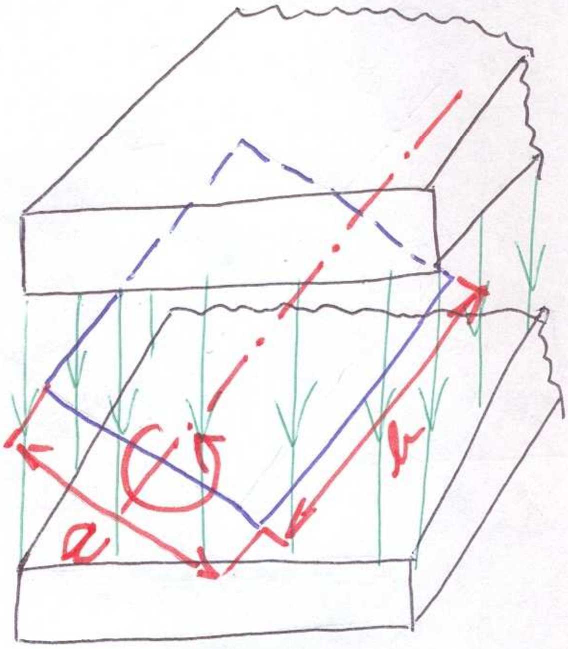
12. Časovo premenné elektromagnetické pole.

Aké najväčšie stredné elektromotorické napätie sa indukuje za pol otáčky obdĺžnikového závitu s rozmermi a= 25 cm , b=30cm okolo osi vedenej stredmi dlhších strán kolmo na magnetické pole, keď sa otáča s frekvenciou f = 1200min^{-1} v magnetickom poly intenzity 0,048.10^{7}Am^{-1}



$a = 25\,cm = 0{,}25\,m$

$b = 30\,cm = 0{,}3\,m$

$f = 1200\,min^{-1} = 20\,s^{-1}$

$H = 0{,}048 \cdot 10^{7}\,A\,m^{-2}$

$\mu_0 = 4\pi \cdot 10^{-7}\,A^{-2}\,kg\,m\,s^{2}$

$U_s = ?$

$B = \mu_0 H = 4\pi \cdot 10^{-7} \cdot 0{,}048 \cdot 10^{7}\,A\,m^{-2} = 0{,}602\,T$

$S_0 = a \cdot b = 0{,}25\,m \cdot 0{,}3\,m = 0{,}075\,m^2$

$\Phi = B \cdot \boxed{S} = B\,S_0 \cdot \cos\alpha$

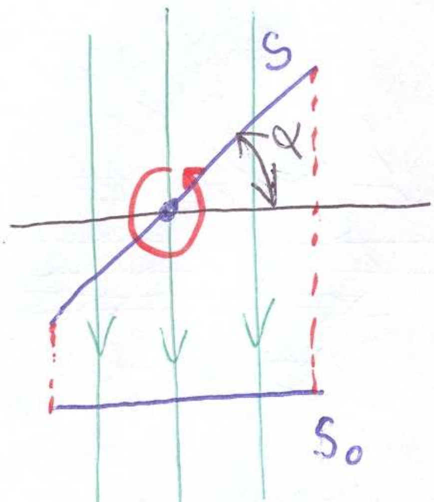
$S = \boxed{S_0 \cos\alpha} = \boxed{S_0 \cos(2\pi f t)}$

$\alpha = f(t)$

$\omega = \boxed{2\pi f}$

$\alpha = \boxed{\omega}\,t = \boxed{2\pi f t}$

$\Phi = B\,S_0 \cos(2\pi f t)$



$$U_i = \frac{-d\Phi}{dt} = \frac{-d}{dt} B S_0 \cos(2\pi f t) =$$

$$= + B S_0 \sin(2\pi f t) \cdot [2\pi f] = \underbrace{\boxed{2\pi f B S_0}}_{U_m} \sin(2\pi f t)$$

$$U_i = U_m \sin(2\pi f t)$$

$$U_m = 2\pi f B S_0 = 2\pi \cdot 20 \cdot 0{,}602 \cdot 0{,}075 = 5{,}68\,V$$

$$U_s = 0{,}637 \cdot U_m = \underline{\underline{3{,}6\,V}}$$

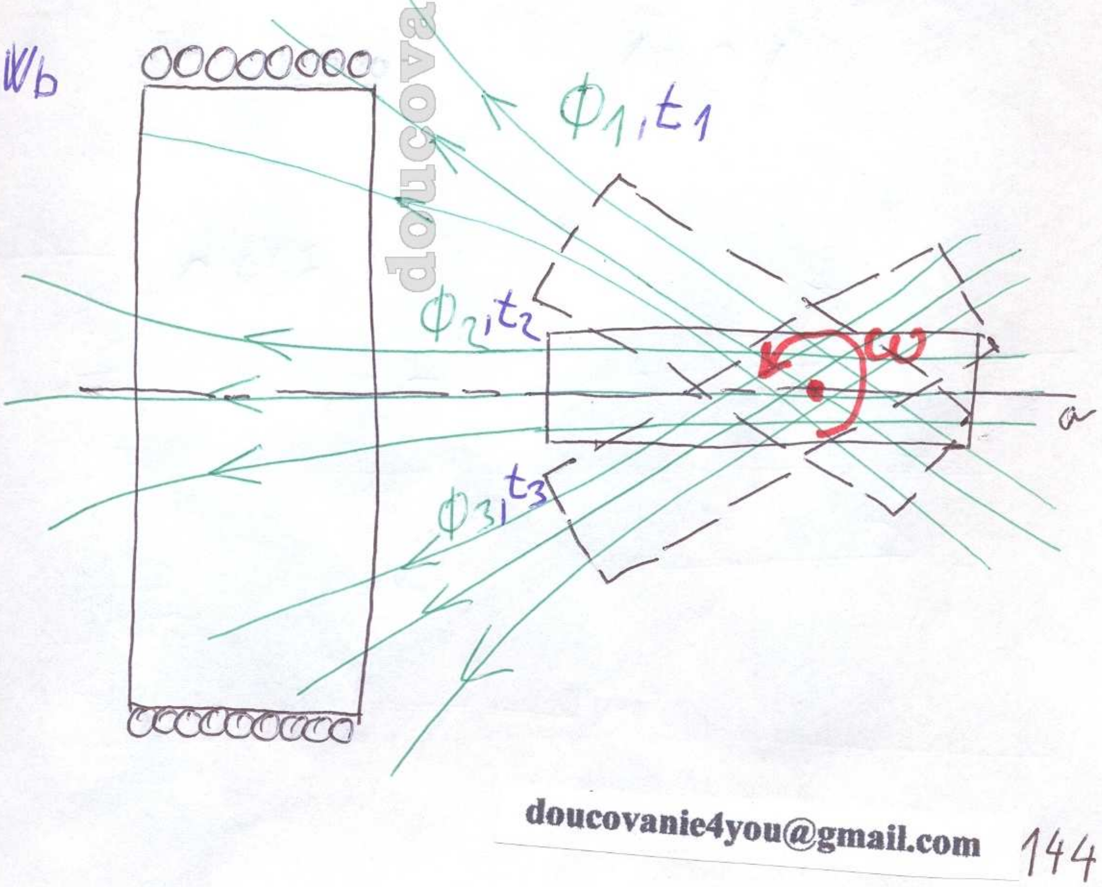
Tyčový magnet s indukčným tokom Φ = 0,0015Wb cez svoj koncový prierez sa otáča okolo zvislej osi tak , že indukčný tok pretína cievku s počtom závitov z= 10000. Jednu polovicu otáčky vykoná magnet za čas t = 0,02s. Vypočítajte strednú hodnotu elektromotorického napätia indukovaného v cievke !

$\Phi_0 = 0{,}0015\,Wb$

$z = 10^4$

$t_{1/2} = 0{,}02\,s$

$U_s = ?$



$$U_i = -z\frac{d\phi}{dt} = -z\frac{d}{dt}\left(\boxed{\phi_0 \cos(157t)}\right) =$$

$$= \underbrace{z\phi_0 157}_{U_m} \sin(157t)$$

$$\phi_t = \boxed{\phi_0 \cos \boxed{\alpha}}$$

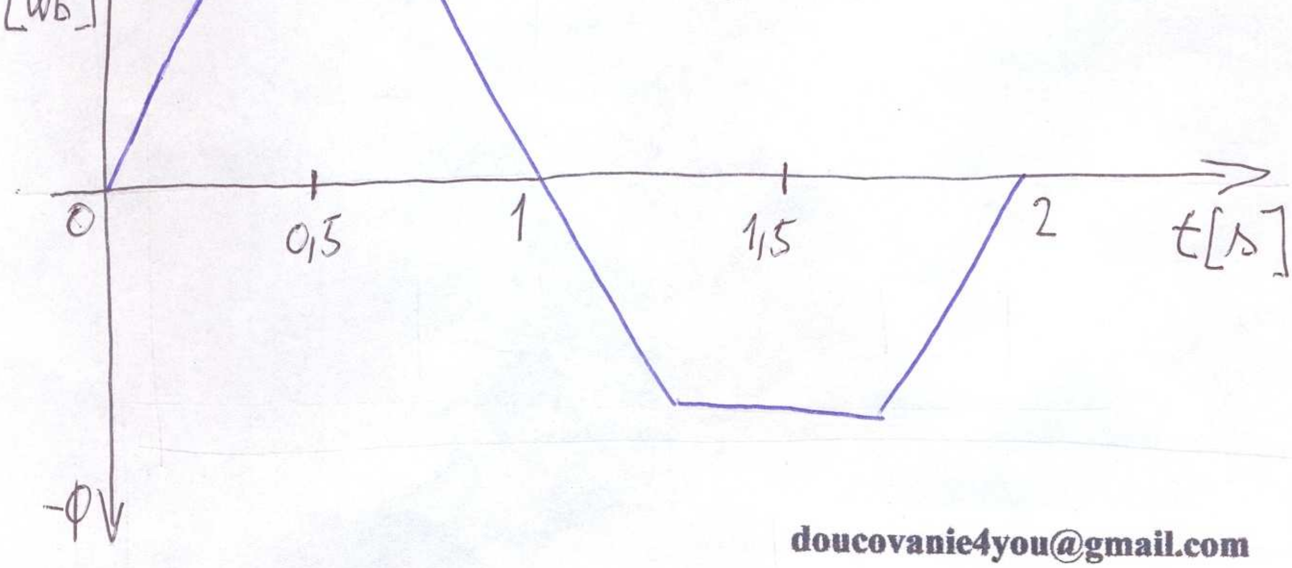
$$\alpha = \boxed{\omega t} = \boxed{314t}$$

$$\omega = 2\pi \boxed{f} = 2\pi\frac{1}{T} = 2\pi\cdot\frac{1}{2\cdot 0{,}02} = \boxed{157}\, s^{-1}$$

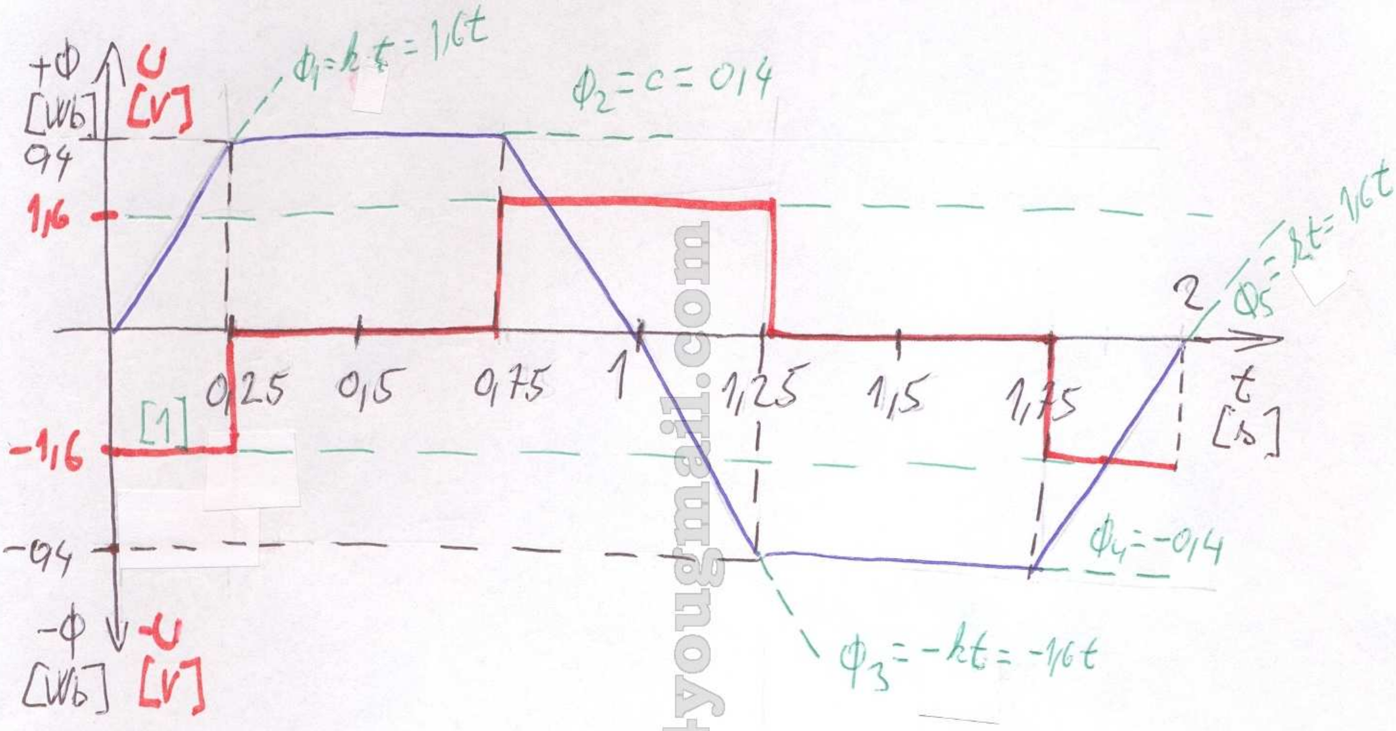
$$f = \boxed{\frac{1}{T}}$$

$$U_m = z\phi_0 157 = 10^4 \cdot 0{,}0015 \cdot 157 = 2355\,V$$

$$U_s = 0{,}637\cdot U_m = 0{,}637\cdot 2355 = \underline{\underline{1500\,V}}$$



Indukčný tok Φ cez vodič tvaru kružnice závisí od času podľa obrázka. Zostrojte závislosť Ui=f(t)



[1] $U_{i_1} = -\frac{d\Phi_1}{dt} = -\frac{d}{dt}(k.t) = -\frac{d}{dt}(1,6t) = \underline{\underline{-1,6\,V}}$

[2] $U_{i_2} = -\frac{d\Phi_2}{dt} = -\frac{d}{dt}(0,4) = \underline{\underline{0\,V}}$

[3] $U_{i_3} = -\frac{d\Phi_3}{dt} = -\frac{d}{dt}(-1,6t) = \underline{\underline{1,6\,V}}$

[4] $U_{i_4} = -\frac{d\Phi_4}{dt} = -\frac{d}{dt}(-0,4) = \underline{\underline{0\,V}}$

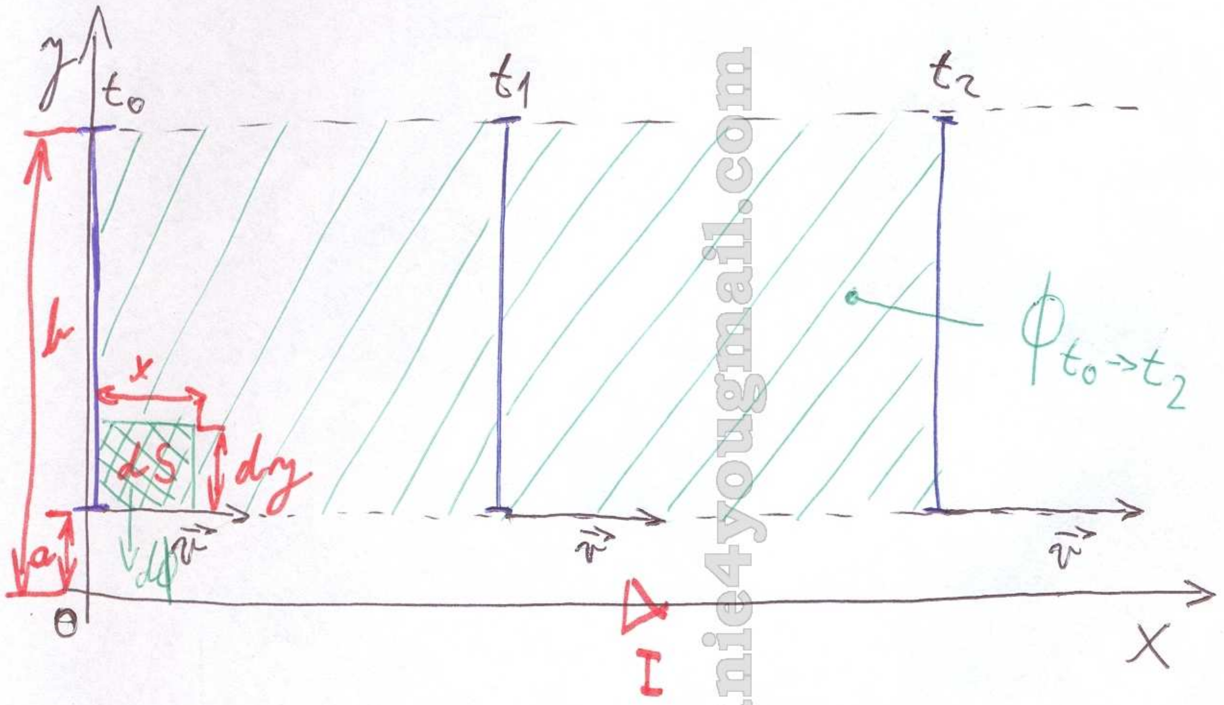
[5] $U_{i_5} = -\frac{d\Phi_5}{dt} = -\frac{d}{dt}(1,6t) = \underline{\underline{-1,6\,V}}$

$k = \frac{0,4}{0,25} = 1,6$

Kovová tyč sa pohybuje stálou rýchlosťou $v = 2ms^{-1}$, rovnobežne s dlhým priamym vodičom, ktorým preteká prúd $I = 40A$. vypočítajte indukované elektromotorické napätie v tyči.

$\mu_0 = 4\pi \cdot 10^{-7} A^{-2} kg\, m\, s^{-2}$ $I = 40A$ $v = 2ms^{-1}$

$b = 100\ cm = 0{,}1\ m$ $a = 10\ cm = 0{,}1\ m$



$$d\phi = B\,dS = B\,v\,t\,dy = \frac{\mu I}{2\pi y} v\,t\,dy$$

$$x = v \cdot t$$

$$dS = x \cdot dy = v \cdot t\,dy$$

$$\boxed{\phi = f(t)}\ !$$

$$\phi = \int_0^l \frac{\mu I v t}{2\pi} \frac{1}{a+y}\, dy = \frac{\mu I v t}{2\pi}\Big[\ln|a+y|\Big]_0^l =$$

KONŠTANTA

$$= \frac{\mu I v t}{2\pi} \ln(a+l) - \ln(a+0) =$$

$$= \frac{\mu I v t}{2\pi} \ln \frac{a+l}{a} = \boxed{\phi}$$

$$\phi = f(t)$$

$$l = b - a = 1 - 0{,}1 = 0{,}9\,m$$

$$U_i = -\frac{d\boxed{\phi}}{dt} = -\frac{d}{dt}\left(\frac{\mu I v t}{2\pi} \ln \frac{a+l}{a}\right) =$$

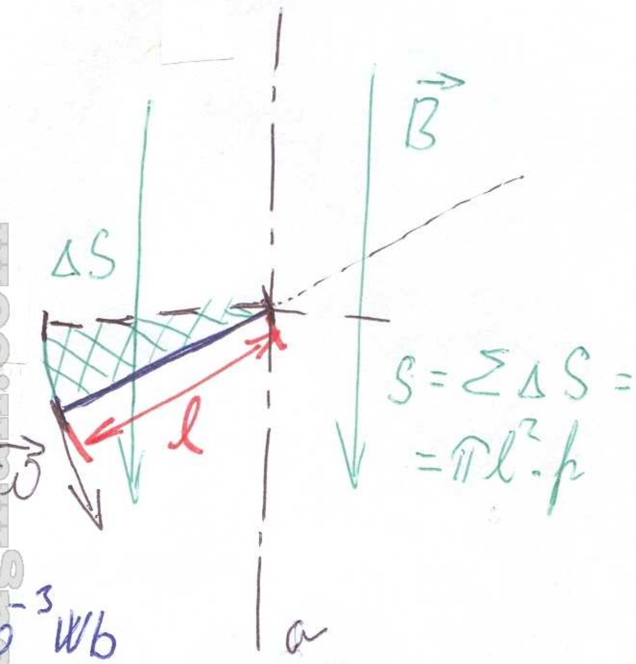
$$= -\frac{\mu I v}{2\pi} \ln \frac{a+l}{a} \frac{d}{dt} t =$$

KONŠTANTA

$$= -\frac{\mu I v}{2\pi} \ln \frac{a+l}{a} = -\frac{4\pi \cdot 10^{-7} \cdot 40 \cdot 2}{2\pi} \ln \frac{0{,}1+0{,}9}{0{,}1} =$$

$$= \underline{-3{,}68 \cdot 10^{-5}\,V}$$

V homogénnom magnetickom poli s indukciou B = 0,2 T sa v rovine kolmej na B otáča vodivá tyč s dĺžkou l=10cm. Os otáčania je kolmá na tyč a prechádza koncovým bodom tyče. Vypočítajte frekvenciu otáčania tyče, keď sa v nej indukuje elektromotorické napätie hodnoty Ui = 0,628V !



$B = 0{,}2\,T$

$l = 10\,cm = 0{,}1\,m$

$U_i = 0{,}628\,V$

$f = ?$

$S_0 = \pi l^2 = 0{,}031\,m^2$

$\phi_0 = B \cdot S = 0{,}2 \cdot 0{,}031 = 6{,}2 \cdot 10^{-3}\,Wb$

$\phi = f \cdot \phi_0$

$U_i = -\frac{\Delta \phi}{\Delta t} \overset{1s}{=} - \qquad \Rightarrow f = \left| \frac{-U_i}{\phi_0} \right|$

$f = \left| -\frac{U_i}{\phi_0} \right| = \left| \frac{-U_i}{B \cdot \pi l^2} \right| = \left| \frac{-0{,}628}{0{,}2 \cdot \pi \cdot 0{,}1^2} \right| = 101{,}3\,Hz$

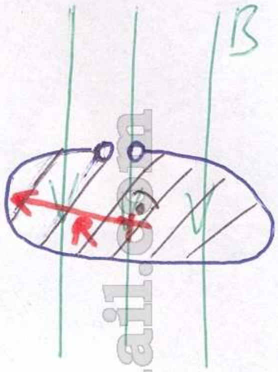
Kruhový vodič polomeru R je v pokoji v magnetickom poli v polohe kolmej na indukčné čiary. Indukcia B magnetického poľa sa v čase lineárne zmenšuje. V časovom okamžiku t=0 indukcia $B=B_0$, v časovom okamžiku $t=t_1$ je $B = 0$. Aké elektromotorické napätie sa indukuje vo vodiči ?

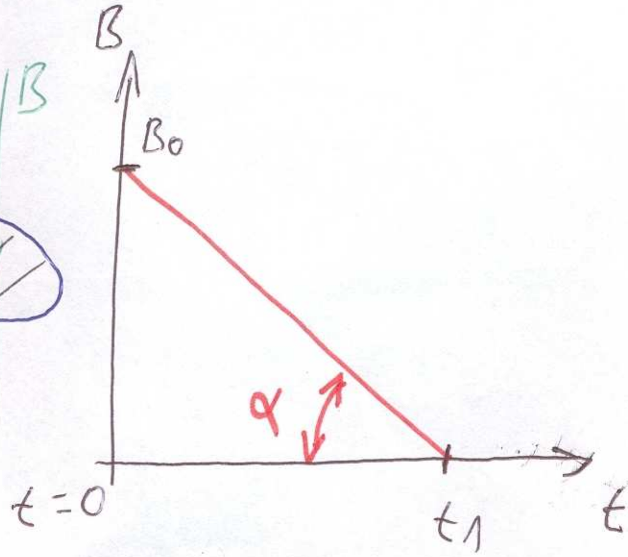
R

$t=0 \quad B=B_0$

$t=t_1 \quad B=0$

$U_i = ?$





$$B=-kt+B_0=-\frac{B_0}{t_1}t+B_0=B_0\left(1-\frac{t}{t_1}\right)$$

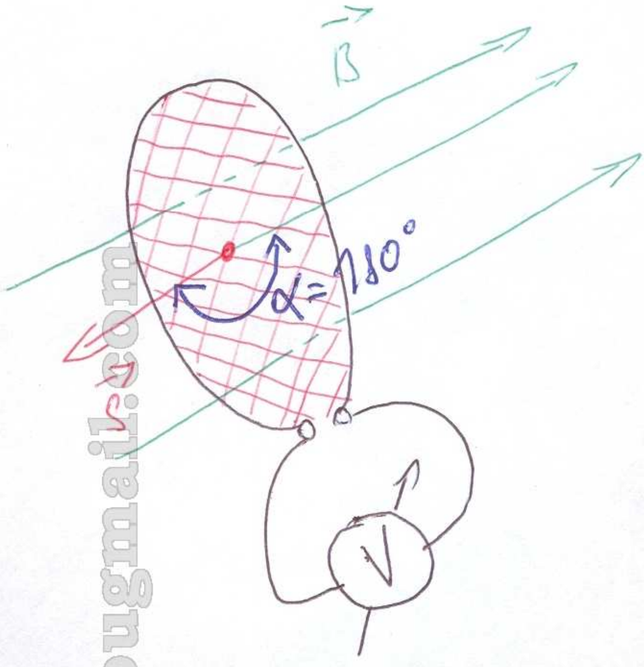
$$k=\frac{B_0}{t_1}=\operatorname{tg}\alpha$$

$$\Phi=BS=B_0\left(1-\frac{t}{t_1}\right)\pi R^2=B_0\pi R^2-\frac{B_0\pi R^2}{t_1}\cdot t$$

$$U_i=-\frac{d\Phi}{dt}=-\frac{d}{dt}\left(B_0\pi R^2-\frac{B_0\pi R^2}{t_1}t\right)=$$

$$\boxed{=\frac{B_0\pi R^2}{t_1}=U_i}$$

Aké maximálne napätie sa indukuje v kruhovom vodiči polomeru R = 5 cm, ktorý je uložený v homogénnom magnetickom poli $B_o = 0{,}5T$ tak , že vektory magnetickej indukcie a plochy zvierajú uhol 180^0 a B sa v čase mení podľa vzťahu $B = B_o \sin(\omega t)$, kde $\omega = 314ms^{-1}$?



$R = 5cm = 0{,}05m$

$B_0 = 0{,}5\,T$

$\alpha = 180°$

$B = B_0 \sin(\omega t)$

$\omega = 314\,s^{-1}$

$B_0^* = B.\cos\alpha$

$B = B_0 \sin\omega t$

$B^* = (B_0 \sin\omega t)\cos\alpha$ $\qquad \phi = B.\,S\cos\alpha = (B_0 \sin\omega t).\,S\cos\alpha$

$$U_i = -\frac{d\phi}{dt} = -\frac{d}{dt}(S B_0 \sin\omega t) = \omega S B_0 \cos\omega t$$

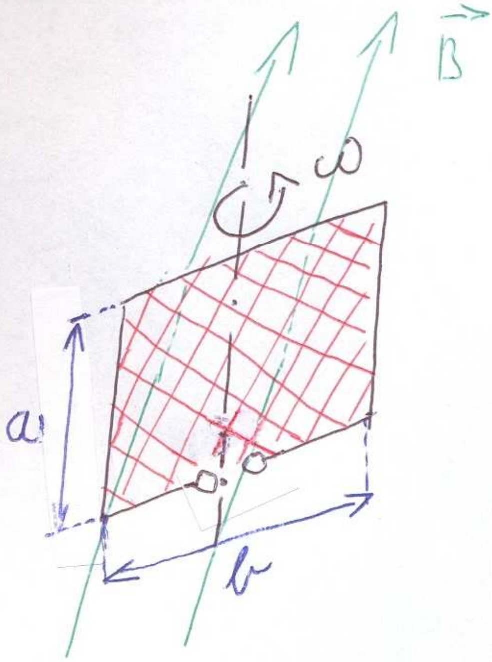
$\omega S B_0 \to U_m$

$U_m = -\omega S B_0$

$U_m^* = -\omega S B_0 \cos\alpha = -314.\pi.(0{,}05)^2.\,0{,}5.\cos 180°$

$U_m = 1{,}23V$

Cievka tvaru obdĺžnika zo stranami a=2cm , b=2,5cm a 100 závitmi sa rovnomerne otáča v homogénnom magnetickom poli s indukciou T = 0,1 T okolo osi kolmej na smer poľa, tak že sa v nej indukuje elektromotorické napätie s amplitúdou U_o = 1,57V. Aká je uhlová rýchlosť otáčania cievky ?



$a = 2\,cm = 0{,}02\,m$

$b = 2{,}5\,cm = 0{,}025\,m$

$N = 100$

$B = 0{,}1\,T$

$U_0 = 1{,}57\,V = U_m$

$\omega = ?$

$\phi = B \cdot S \cos(\omega t)$

$$U_i = -N\frac{d\phi}{dt} = -N\frac{d}{dt}(B \cdot S \cos \omega t)$$

$$U_i = +N\omega B S \sin(\omega t)$$

$U_m = U_0$

$$U_m = N\omega B S \Rightarrow \omega = \frac{U_m}{NBS}$$

$$\omega = \frac{1{,}57}{100 \cdot 0{,}1 \cdot 0{,}02 \cdot 0{,}025} = \underline{\underline{314\,s^{-1}}}$$

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13. Kvantové procesy

Teplota wolfrámovej špirály v **25- wattovej** žiarovke je **T = 2450 K**. Pomer jej celkovej intenzity vyžarovania a vyžarovania absolútne čierneho telesa je **α = 0,3**. Určte veľkosť **S** plochy tejto špirály!

$T = 2450\,K$

$P = 25\,W$

$\alpha = 0{,}3$

$\sigma = 5{,}67 \cdot 10^{-8}\,J\,m^{-2}\,s^{-1}\,K^{-4}$

$S = ?$

$P = S\sigma T^4 \alpha$

$$S = \frac{P}{\sigma T^4 \alpha} = \frac{25}{5{,}67 \cdot 10^{-8} \cdot 2450^4 \cdot 0{,}3} = 4{,}07 \cdot 10^{-5}\,m^2$$

Slnečná konštanta K je množstvo slnečnej energie vyžiarenej slnkom za jednotku času cez jednotku plochy kolmo k slnečným lúčom. Vypočítajte hodnotu slnečnej konštanty K na povrchu Zeme. Slnko považujeme za absolútne čierne teleso s teplotou **T = 5800K** Vzdialenosť Slnka od Zeme je **$d = 149{,}59.10^6$ km** . Polomer Slnka **Rs = 695990 km, $\sigma = 5{,}67.10^{-8}\,Jm^{-2}s^{-1}K^{-4}$**

$T = 5800\,K$

$d = 149{,}59 \cdot 10^6\,km = 149{,}59 \cdot 10^9\,m$

$R_S = 695990\,km = 695990\,000\,m$

$\sigma = 5{,}67 \cdot 10^{-8}\,J\,m^{-2}\,s^{-1}\,K^{-4}$

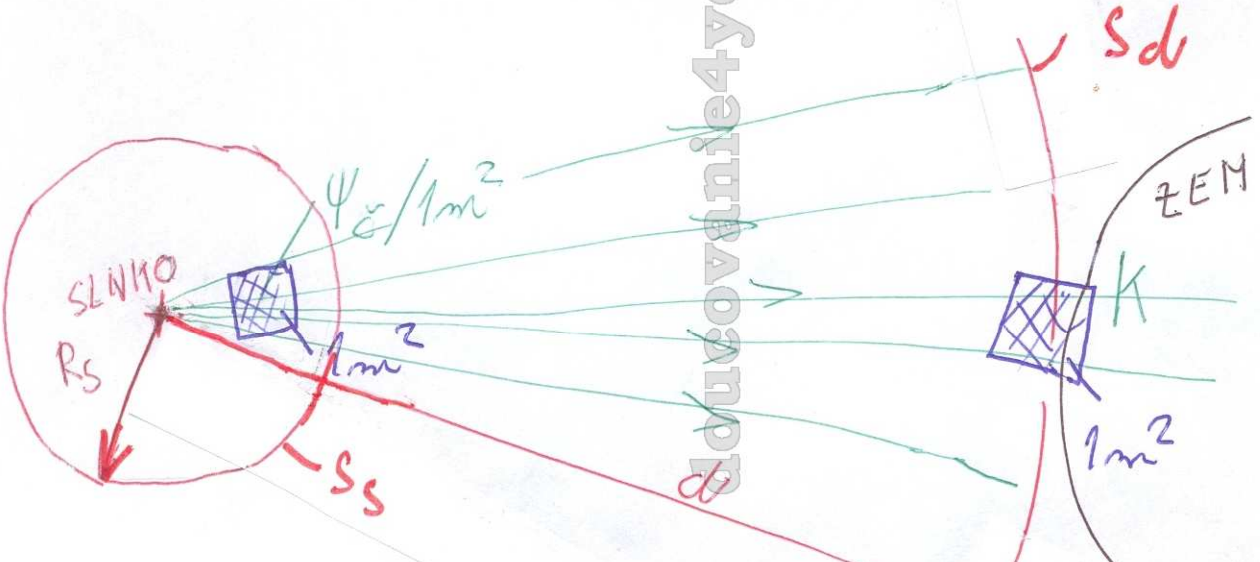
$K = ?$

$$\Psi_{\check{c}} = \sigma T^4 = 5{,}67 \cdot 10^{-8} \cdot 5800^4 = 64{,}16 \cdot 10^6\, W m^{-2}$$

$$P = \Psi_{\check{c}} \cdot S = 64{,}16 \cdot 10^6 \cdot \underbrace{4\pi R_S^2}_{S_S} = 3{,}9 \cdot 10^{26}\, W$$

$$S_d = 4\pi d^2 = 2{,}81 \cdot 10^{23}\, m^2$$

$$K = \frac{P}{S_d} = \frac{3{,}9 \cdot 10^{26}\, W}{2{,}81 \cdot 10^{23}\, m^2} = \underline{1{,}38 \cdot 10^3\, W m^{-2}}$$



Energia vyžiarená rozžeraveným kovovým povrchom za jednotku času je **P´ = 0,67 kW**. Teplota povrchu je 1500K , veľkosť plochy **S = 10 cm²** . Aká energia by sa vyžiarila týmto povrchom za jednotku času, keby bol absolútne čierny ? Nájdite pomer intenzít α vyžarovania tohto povrchu a absolútne čierneho telesa pri danej teplote !

$\sigma = 5{,}67.10^{-8}\ Jm^{-2}s^{-1}K^{-4}$

$P' = 0{,}67\,kW = 0{,}67.10^3\,W$

$T = 2500\,K$

$S = 10\,cm^2 = 10.10^{-4}\,m^2$

$P = ?\quad \alpha = ?$

$P = \sigma T^4 S = \Psi$

$$\alpha = \frac{P'}{P} = \frac{P'}{\sigma T^4 S} = \frac{0{,}67.10^3}{5{,}67.10^{-8}.10.10^{-4}.2500^4}$$

$\alpha = 0{,}3025$

$P = \sigma T^4 S = 5{,}67.10^{-8}.10.10^{-4}.2500^4 = 2214\,W$

Obvykle, sa udáva, že stredná hodnota energie, ktorú vyžiari 1 cm^2 zemského povrchu za jednu minútu je 0,54J. Akú teplotu by malo mať absolútne čierne teleso ktoré by vyžarovalo rovnaké množstvo energie. **$\sigma = 5{,}67.10^{-8}\ Jm^{-2}s^{-1}K^{-4}$**

$S = 1\,cm^2 = 1\cdot 10^{-4}\,m^2$

$t = 1\,min = 60\,s$

$E = 0{,}54\,J$

$\sigma = 5{,}67\cdot 10^{-8}\,J\,m^{-2}\,s^{-1}\,K^{-4}$

$T = ?$

$$P = \frac{E}{t} = \frac{0{,}54\,J}{60\,s} = 9\cdot 10^{-3}\,W$$

$$P = \sigma T^4 S \qquad T = \sqrt[4]{\frac{P}{\sigma S}}$$

$$T = \sqrt[4]{\frac{9\cdot 10^{-3}}{5{,}67\cdot 10^{-8}\cdot 1\cdot 10^{-4}}} = 199{,}6\,K$$

Teleso zohriate na teplotu **2500 K** necháme postupne chladnúť. Vlnová dĺžka svetla na ktorú pripadá relatívne najviac energie v spektre žiarenia toho telesa sa pritom zmení o **0,8um** . Vypočítate na akú teplotu sa teleso ochladilo, ak predpokladáme , že žiari ako absolútne čierne !
b = 289.10^{-5} mK.

$T_1 = 2500K$

$\Delta\lambda = 0{,}8\,\mu m = 0{,}8 \cdot 10^{-6}\,m$

$b = 289 \cdot 10^{-5}\,m\,K$

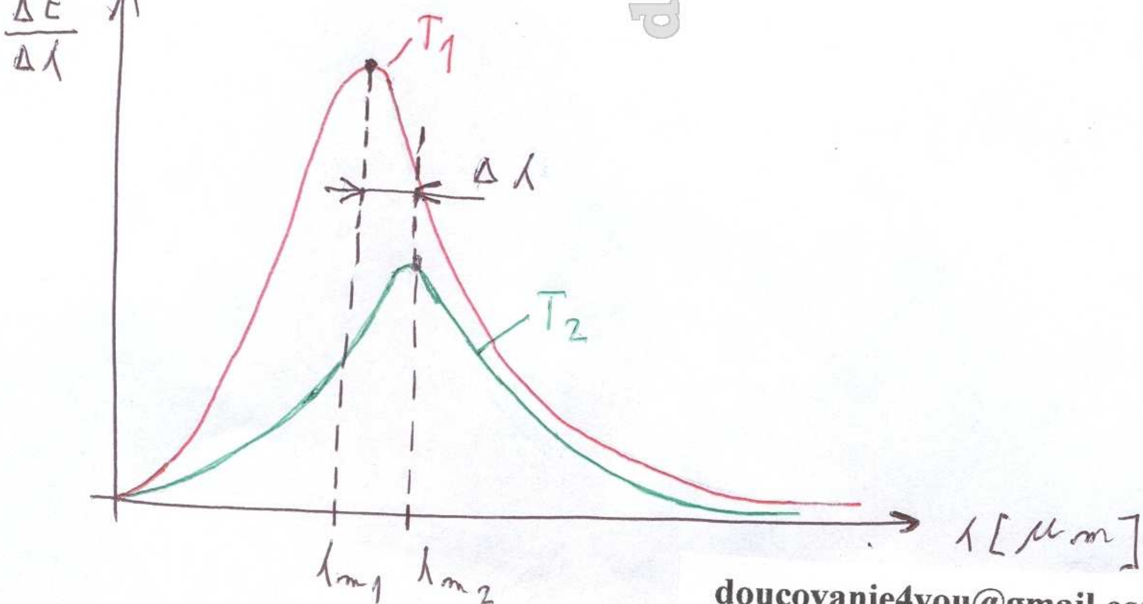
$\Delta T = ?$

$\lambda_{m_1} \cdot T_1 = b$

$$\lambda_{m_1} = \frac{b}{T_1} = \frac{289 \cdot 10^{-5}}{2500} = 1{,}15 \cdot 10^{-6}\,m$$

$$\lambda_{m_2} = \lambda_{m_1} + \Delta\lambda = 1{,}15 \cdot 10^{-6} + 0{,}8 \cdot 10^{-6} = 1{,}95 \cdot 10^{-6}$$

$$T_2 = \frac{b}{\lambda_{m_2}} = \frac{289 \cdot 10^{-5}}{1{,}95 \cdot 10^{-6}} = \underline{1482K}$$



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Na 1 cm^2 zemského povrchu dopadá zo Slnka asi 8,12 J energie za minútu. Vypočítajte aká je povrchová teplota Slnka za predpokladu , že Slnko žiari ako absolútne čierne teleso. Vzdialenosť Zeme od Slnka je **$d = 149{,}59.10^6$ km** polomer Slnka je **$R_s = 695990$ km** , **$\sigma = 5{,}67.10^{-8}\ Jm^{-2}s^{-1}K^{-4}$**

$S = 1\,cm^2 = 1.10^{-4}\,m^2$

$E = 8{,}12\ J$

$t = 1\,min = 60\,s$

$d = 149{,}59.10^6\,km = 149{,}59.10^9\,m$

$R_S = 695990\,km = 695{,}9.10^6\,m$

$\sigma = 5{,}67.10^{-8}\ J\,m^{-2}s^{-1}K^{-4}$

$T = ?$

$S_1 = 4\pi d^2 = 4\pi.(149{,}59.10^9)^2 = 2{,}81.10^{23}\,m^2$

$P = \frac{E}{t} = \frac{8{,}12}{60} = 0{,}13\,W\,cm^{-2}$

$P_{1m^2} = 10000.P = 10000.0{,}13 = 1300\,W\,m^{-2}$

$P_{S_1} = S_1.P_{1m^2} = 2{,}81.10^{23}.1300 = 3{,}65.10^{26}\,W$

$S_2 = 4\pi R_S^2 = 4\pi.(695{,}9.10^6)^2 = 6{,}08.10^{18}\,m^2$

$P^*_{1m^2} = \frac{3{,}65.10^{26}}{6{,}08.10^{18}} = 60\,MW\,m^{-2}$

$$P = \sigma T^4$$

$$T = \sqrt[4]{\frac{P^*_{1m^2}}{\sigma}} = \sqrt[4]{\frac{60.10^6}{5{,}67.10^{-8}}} = \underline{\underline{5703{,}5\,K}}$$

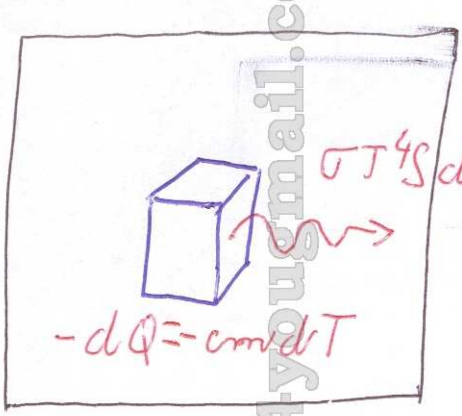
Do čiernej tenkostennej nádoby tvaru kocky sme naliali 1kg vody teploty 50^0 C a voda vyplnila celý objem nádoby. Vypočítajte za aký čas sa voda ochladí na teplotu 10^0C, keď je umiestnená v čiernej dutine , ktorej steny majú teplotu blízku absolútnej nule !
$\sigma = 5{,}67.10^{-8}\ Jm^{-2}s^{-1}K^{-4}$ $C_v = 4186\ J\ kg^{-1}K^{-1}$.

$m = 1\,kg$

$t_1 = 50\,°C$

$t_2 = 10\,°C$

$t = ?\,[s]$



$\sigma T^4 S\, dT = +dQ$

$-dQ = -cm\,dT$

$|+dQ| = |-dQ|$

PRÍRASTOK TEPLOTY STENY SA ROVNÁ POKLESU TEPLOTY VODY

$T_1 = 50 + 273{,}15 = 323{,}15\,K$

$T_2 = 10 + 273{,}15 = 283{,}15\,K$

$V = a^3 \quad a = \sqrt[3]{V} = \sqrt[3]{1.10^{-3}} = 0{,}1\,m$

$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho} = \frac{1}{1000} = 1.10^{-3}\,m^3$

$S = 6a^2 = 6.(0{,}1)^2 = 0{,}06\,m^2$

$P = \sigma T^4 S$

$dQ = c\,m\,dT$

$dT = \dfrac{dQ}{c\,m}$

$-dQ = \sigma T^4 S\,dt$

$\boxed{-c\,m\,dT = \sigma T^4 S\,dt}$! DIFERENCIÁLNA ROVNICA

$-\dfrac{1}{T^4}\,dT = \dfrac{\sigma S}{c\,m}\,dt \quad \Big/ \int$

$-\displaystyle\int_{T_1}^{T_2} \frac{1}{T^4}\,dT = \frac{\sigma S}{c\,m} \int_0^t dt$

$+\displaystyle\int_{T_2}^{T_1} \frac{1}{T^4}\,dT = \frac{\sigma S}{c\,m} \int_0^t dt$

$\left[\dfrac{T^{-3}}{-3}\right]_{T_1}^{T_2} = \dfrac{\sigma S}{c\,m}\Big[t\Big]_0^t$

$\dfrac{T_2^{-3}}{-3} - \dfrac{T_1^{-3}}{-3} = \dfrac{\sigma S}{c\,m}\,t$

$t = \left(-\dfrac{1}{3T_2^{\,3}} + \dfrac{1}{3T_1^{\,3}}\right) \cdot \dfrac{c\,m}{\sigma S} = 5916{,}3\,s = \underline{\underline{1h\ 38\,min\ 36\,s}}$

Vpočítajte, aký prúd by mal pretekať kovovým vláknom priemeru d = 0,1mm, ktoré je vo vyčerpanej banke, aby sa jeho teplota udržiavala na stálej teplote **T = 2500 K** ! Predpokladáme, že vlákno vyžaruje energiu ako absolútne čierne teleso. Tepelné straty spojená s vedením tepla zanedbajte. Merný odpor vlákna je **$\rho = 2{,}5 . 10^{-4}$ cm** , **$\sigma = 5{,}67 . 10^{-8}\ Jm^{-2}s^{-1}K^{-4}$**

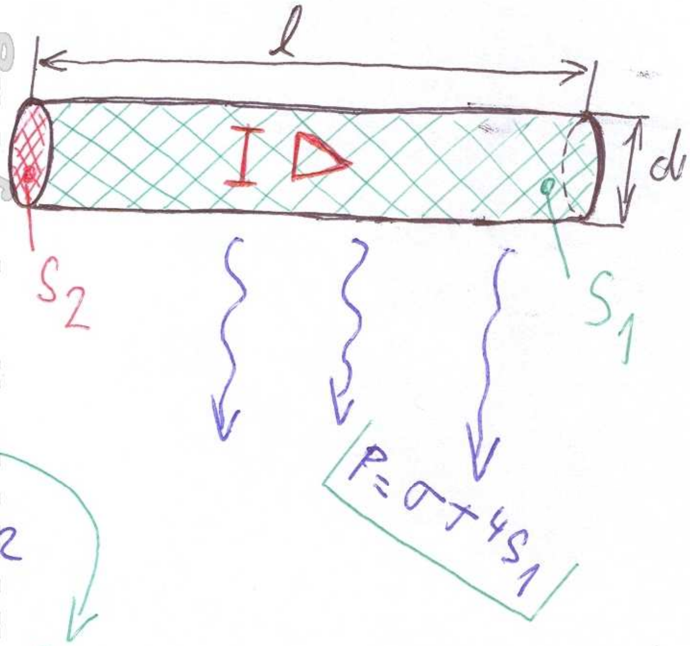
$d = 0{,}1\,mm = 0{,}1 \cdot 10^{-3}\,m$

$T = 2500\,K$

$\rho = 2{,}5 \cdot 10^{-4}\,\Omega\,cm = 2{,}5 \cdot 10^{-6}\,\Omega\,m$

$\sigma = 5{,}67 \cdot 10^{-8}\,J m^{-2} s^{-1} K^{-4}$

$I = ?\,[A]$



$$P = RI^2$$

$$\sigma T^4 S_1 = RI^2$$

$$\sigma T^4 (\pi d l) = \rho \frac{l}{S_2} I^2$$

$$\sigma T^4 (\pi d l) = \rho \frac{l}{\underbrace{\pi \frac{d^2}{4}}_{S_2}} I^2$$

$$I^2 = \frac{\pi d^2}{4 \rho \cancel{l}} \sigma T^4 \pi d \cancel{l} = \frac{\sigma T^4 \pi^2 d^3}{4\rho}$$

$$I = \sqrt[2]{\frac{\sigma T^4 \pi^2 d^3}{4\rho}} = \underline{\underline{1{,}47 A}}$$

14. Elektrónový obal a jadro atómu.

Vypočítajte obvodovú rýchlosť a polomer dráhy elektrónu na tretej kvantovej dráhe v Bohrovom modeli atómu vodíka !
($h = 6{,}626 .10^{-34}$ Js , $m_e = 9{,}1.10^{-31}$ kg , $e = 1{,}602.10^{-19}$As , $\varepsilon_o = 8{,}854.10^{-12}\ m^{-3}kg^{-1}s^4A^2$)

$n = 3$

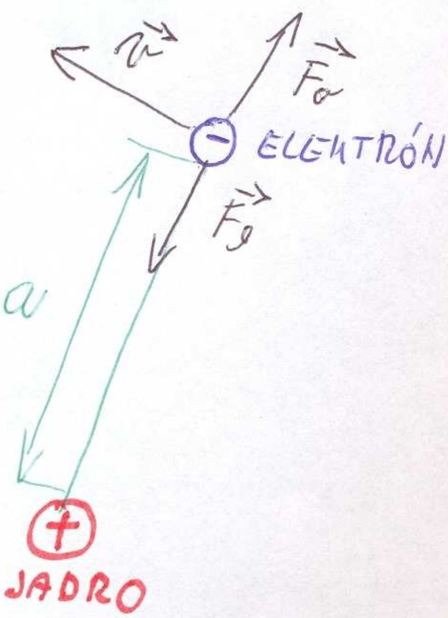
$h = 6{,}626 \cdot 10^{-34}\ Js$

$m_e = 9{,}1 \cdot 10^{-31}\ kg$

$e = 1{,}602 \cdot 10^{-19}\ C$

$\varepsilon_0 = 8{,}854 \cdot 10^{-12}\ m^{-3} kg^{-1} s^4 A^2$

$v, r = ?$



$$\text{I. } nh = 2\pi a m v \Rightarrow v = \frac{nh}{2\pi a m}$$

$$\text{II. } \frac{v^2}{a} m = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{a^2}$$

$$|\vec{F}_o| = |-\vec{F}_e|$$

$$\frac{\left(\frac{nh}{2\pi a m}\right)^2}{a} \cdot m = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{a^2}$$

$$a = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2} = \frac{3^2 (6{,}6 \cdot 10^{-34})^2 \cdot 8{,}8 \cdot 10^{-12}}{\pi \cdot 9{,}1 \cdot 10^{-31} (1{,}6 \cdot 10^{-19})^2} = 4{,}72 \cdot 10^{-10}\ m$$

$$v = \frac{nh}{2\pi a m} = \frac{3 \cdot 6{,}6 \cdot 10^{-34}}{2\pi \; 4{,}72 \cdot 10^{-10} \cdot 9{,}1 \cdot 10^{-31}}$$

$$v = 761538{,}4 \, m s^{-1} = \underline{761{,}53 \, km s^{-1}}$$

Aká je perióda obiehania elektrónu na tretej kvantovej dráhe v Bohrovom modeli vodíkového atómu.
($h = 6{,}626 \cdot 10^{-34}$ Js , $m_e = 9{,}1 \cdot 10^{-31}$ kg , $e = 1{,}602 \cdot 10^{-19}$As , $\varepsilon_o = 8{,}854 \cdot 10^{-12}$ $m^{-3}kg^{-1}s^4A^2$)

$n = 3$

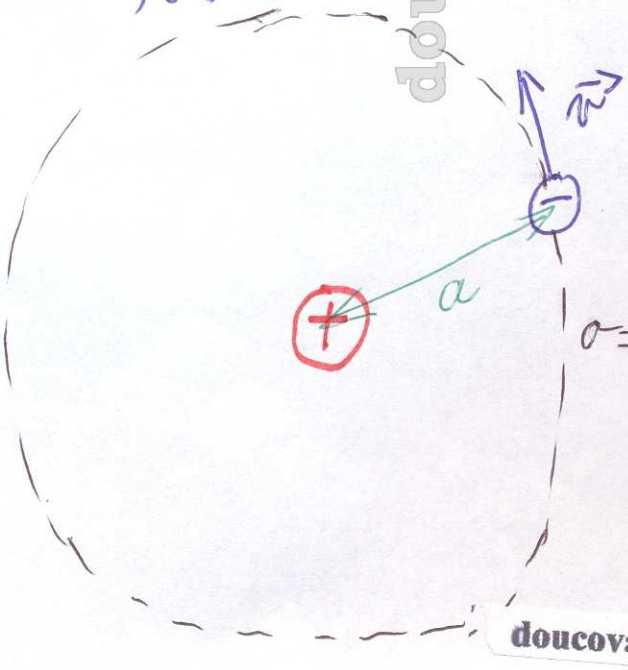
$v = 761 \cdot 10^3 \, m s^{-1}$

$a = 4{,}72 \cdot 10^{-10} \, m$

$T = ?$

$$T = \frac{s}{v} = \frac{2\pi a}{v} = \frac{2\pi \cdot 4{,}72 \cdot 10^{-10}}{761 \cdot 10^3} = 3{,}89 \cdot 10^{-15} s$$

$$f = \frac{1}{T} = \frac{1}{3{,}89 \cdot 10^{-15}} = 2{,}56 \cdot 10^{14} Hz = 256 \, THz$$



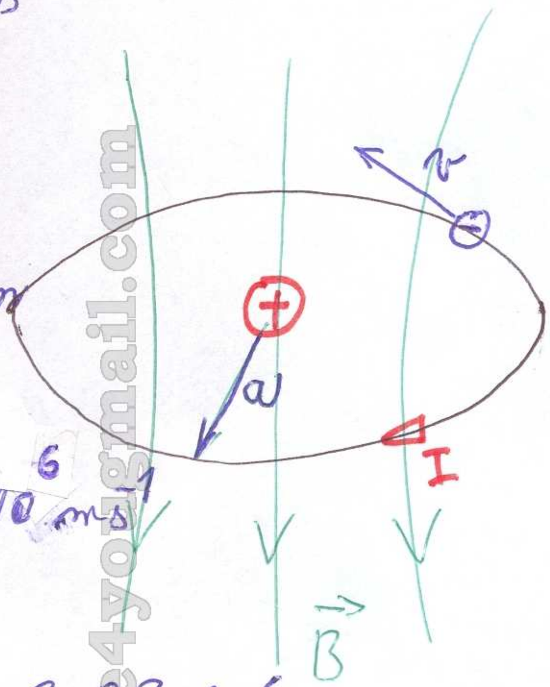
$o = 2\pi r = 2\pi a = s$

Vypočítajte indukciu magnetického poľa, ktoré je vytvárané elektrónom obiehajúcim v Bohrovom modeli atómu vodíka po prvej dovolenej dráhe, v strede tejto dráhy.
($h = 6{,}626 .10^{-34}$ Js , $m_e = 9{,}1.10^{-31}$ kg , $e = 1{,}602.10^{-19}$As , $\varepsilon_o = 8{,}854.10^{-12}$ $m^{-3}kg^{-1}s^4A^2$,
$\mu_0 = 4\pi .10^{-7} A^{-2}kg\, ms^{-2}$)

$n = 2$

$\mu_0 = 4\pi \cdot 10^{-7} A^{-2} kg\, m\, s^{-2}$

$B = ?$



$$a = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2} = 5{,}2 \cdot 10^{-11} m$$

$$v = \frac{nh}{2\pi a m} = 2{,}22 \cdot 10^{6} ms^{-1}$$

$$f = \frac{v}{s} = \frac{v}{2\pi a} = \frac{2{,}22 \cdot 10^6}{2\pi \cdot 5{,}2 \cdot 10^{-11}} = 6{,}79 \cdot 10^{-15} Hz$$

$$I = e \cdot f = 1{,}08 \cdot 10^{-3} A$$

$$B = \frac{\mu_0 I}{2a} = \frac{4\pi \cdot 10^{-7} \cdot 1{,}08 \cdot 10^{-3}}{2 \cdot 5{,}2 \cdot 10^{-11}} = \underline{13\,T}$$

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Vypočítajte celkovú energiu elektrónu na druhej kvantovej dráhe v Bohrovom modeli atómu vodíka.
($h = 6{,}626 \,.10^{-34}$ Js , $m_e = 9{,}1.10^{-31}$ kg , $e = 1{,}602.10^{-19}$As , $\varepsilon_0 = 8{,}854.10^{-12}$ $m^{-3}kg^{-1}s^4A^2$)

$n = 2$

$W_c = ?$

$W_c = W - W_p$

$$\frac{v^2}{a} m = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{a^2} \Big/ \cdot \frac{1}{2}$$

$$|\vec{F_0}| = |-\vec{F_e}|$$

$$\frac{1}{2} m v^2 = \frac{1}{2} \frac{1}{4\pi\varepsilon_0} \frac{e^2}{a}$$

$$W_k = \frac{1}{2} \frac{1}{4} \frac{1}{\pi\varepsilon_0} \frac{e^2}{a} = \frac{(1{,}6 . 10^{-19})^2}{8 . \pi . 8{,}8 . 10^{-12}} = 5{,}55 . 10^{-19} \, J$$

$$W_p = \int_1^2 \vec{F_e} d\vec{r} = \frac{e^2}{4\pi\varepsilon_0} \int_\infty^a \frac{1}{r^2} dr = \frac{e^2}{4\pi\varepsilon_0} \left[-\frac{1}{r} \right]_\infty^a =$$

$$= -\frac{e^2}{4\pi\varepsilon_0} \frac{1}{a} = -\frac{e^4 m}{\varepsilon_0^2 n^2 h^2} \qquad a = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2}$$

$W_p = 1{,}11 \cdot 10^{-18}\,J$

$W_c = W_k - W_p = 5{,}55 \cdot 10^{-19} - 1{,}11 \cdot 10^{-18} = \underline{-5{,}55 \cdot 10^{-19}\,J}$

Aká je vlnová dĺžka svetla, ktorá odpovedá preskokom elektrónov zo šiestej na druhú kvantovú dráhu V Bohrovom modeli atómu vodíka ? Aká vlnová dĺžka prislúcha hrane Balmerovej série.
($R = 1{,}097373 \cdot 10^{7}\,m^{-1}$)

$n = 6 \quad k = 2$

$n > k$

$R = 1{,}097 \cdot 10^{7}\,m^{-1}$

$\lambda_1, \lambda_2 = ?$

$$\frac{1}{\lambda} = \frac{m e^4}{8 \varepsilon_0^2 h^3 c}\left(\frac{1}{k^2} - \frac{1}{n^2}\right) =$$

$$= \frac{9{,}1 \cdot 10^{-31} \cdot (1{,}602 \cdot 10^{-19})^4}{8 \cdot (8{,}85 \cdot 10^{-12})^2 \cdot (6{,}6 \cdot 10^{-34})^3 \cdot 3 \cdot 10^8}\left(\frac{1}{2^2} - \frac{1}{6^2}\right) = \ldots$$

VLNOČET

$$\vec{\nu} = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right) = 1{,}097 \cdot 10^{7}\left(\frac{1}{2^2} - \frac{1}{6^2}\right) = 2{,}37 \cdot 10^{6}\,m^{-1}$$

$$\lambda = \frac{1}{\nu} = \frac{1}{2{,}37 \cdot 10^{6}} = \underline{4{,}2 \cdot 10^{-7}\,m}$$

BALMEROVÁ SÉRIA $(n = 3 \quad k = 2)$ OKRAJ

$$\nu = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right) = 1{,}097 \cdot 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\nu = 1{,}52 \cdot 10^6 \, m^{-1}$$

$$\lambda = \frac{1}{\nu} = \frac{1}{1{,}52 \cdot 10^6} = \underline{6{,}56 \cdot 10^{-7} \, m}$$

BALMEROVÁ SÉRIA ($n = 3$ $k = \infty$) OKRAJ

$$\nu = R\left(\frac{1}{k^2} - \frac{1}{n^2}\right) = 1{,}097 \cdot 10^7 \left(\frac{1}{2^2} - \underbrace{\frac{1}{\infty^2}}_{=0}\right)$$

$$\nu = 2{,}74 \cdot 10^3 \, m^{-1}$$

$$\lambda = \frac{1}{\nu} = \frac{1}{2{,}74 \cdot 10^3} = \underline{3{,}64 \cdot 10^{-7} \, m}$$

Konečným produktom rádioaktívneho rozpadu $^{232}_{90}Th$ je izotop olova $^{208}_{82}Pb$. Vypočítajte koľko α častíc a koľko β častíc sa uvoľní pri tomto rozpade.

$^{232}_{90}Th$

$^{208}_{82}Pb$

$\alpha, \beta = ?$

232 − 208 = 24 NUKLEÓNOV

90 − 82 = 8 PROTÓNOV

1α = 2 PROTÓN + 2 NEUTRÓN

8 PROTÓNOV STAČÍ NA 4α ČASTICE

8 NEUTRÓNOV NA TIETO 4α ČASTÍC

24 − (8+8) = 8 NEUTRÓNOV OSTANE

8 NEUTRÓNOV TREBA NA 2α ČASTICE

2α + 4α = 6α ČASTÍC

4 PROTONY 2+2 NA KAŽDÚ α ČASTICU

4 NEUTRÓNY SA PRETO ZMENIA NA PROTÓNY

UVOĽNÍ SA 4β ČASTÍC

doucovanie4you@gmail.com

Vypočítajte za aký čas sa rozpadne štvrtina pôvodného množstva rádioaktívnej látky ktorej polčas rozpadu je jeden rok.

$T_{1/2} = 1\,rok = 31536000\,s$

$t = ?$

$T_{1/2} = \frac{\ln 2}{\lambda} \Rightarrow \lambda = \frac{\ln 2}{T_{1/2}}$

$N = N_0 e^{-\lambda t}$

$\frac{3}{4} N_0 = N_0 e^{-\lambda t}$

$\frac{3}{4} = e^{-\lambda t} \quad / \ln$

$\ln \frac{3}{4} = \ln e^{-\lambda t}$

$\ln \frac{3}{4} = -\lambda t \ln e$

$\ln \frac{3}{4} = -\frac{\ln 2}{T_{1/2}} t$

$t = -\ln \frac{3}{4} \frac{T_{1/2}}{\ln 2} = -\ln \frac{3}{4} \frac{31536000}{\ln 2} =$

$= 12{,}79 \cdot 10^6\,s = \underline{148{,}11\ DNÍ}$

Za aký čas sa rozpadne z 107 atómov aktínia jeden atóm ak polčas rozpadu aktínia je 13,5 rokov.

$N_0 = 10^7$

$T_{1/2} = 13,5$ ROKOV

$(N_0 - 1) = N_0 e^{-\lambda t}$ $\qquad N = (N_0 - 1)$

$\lambda = \frac{\ln 2}{T_{1/2}}$

$\frac{N_0 - 1}{N_0} = e^{-\lambda t} \; / \ln$

$\ln\left(\frac{N_0 - 1}{N_0}\right) = \ln e^{-\lambda t}$

$\ln\left(\frac{N_0 - 1}{N_0}\right) = -\lambda t \ln e \qquad \ln e = 1$

$$\frac{\ln\left(\frac{N_0 - 1}{N_0}\right)}{-\lambda} = t = \frac{\ln\left(\frac{N_0 - 1}{N_0}\right)}{-\frac{\ln 2}{T_{1/2}}} =$$

$$= \frac{\ln\left(\frac{10^7 - 1}{10^7}\right)}{-\frac{\ln 2}{13,5 \cdot 365 \cdot 24 \cdot 3600}} = \underline{61,34\,s}$$

doucovanie4you@gmail.com

Zaujímavé internetové stránky

www.zachranmezivoty.sk

www.coskolaneuci.sk

www.stopgenocide.sk

www.dtest.cz